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SOME FIXED POINT THEOREMS FOR REICH TYPE CONTRACTION IN GENERALIZED METRIC SPACES

XIAOYAN LV, YUQIANG FENG

Abstract. In this paper, an analogue of Reich type contraction in rectangular b-metric spaces is discussed, which is an answer to Reny George's question [1]. A generalized fixed point theorem of Reich type contraction is proved in b-metric space, which relaxes the contraction condition.

1. Introduction

Some problems, particularly the problem of the convergence of measurable functions with respect to measure lead to a generalization of notion of metric. Using this idea S. Czerwik [2] presented the concept of b-metric space. And he presented generalization of some fixed point theorems of Banach type in b-metric space. Since then, several papers have dealt with fixed point theory for various definitions of contractive mappings in b-metric space (see [3-7],[17-39]). For example, in [8] obtained a Reich type fixed point theorem in b-metric space as follows:

Theorem 1.1[8] Let M be a complete b-metric space with metric $\rho$and let $T : M \to M$ be a function with the following

$$\rho(T(x), T(y)) \leq a\rho(x, T(x)) + b\rho(y, T(y)) + c\rho(x, y)$$

$\forall x, y \in M$, where a, b, c are non-negative real numbers and satisfy $a + s(b + c) < 1$ for $s \geq 1$ then T has a unique fixed point.

On the other hand, Branciari [9] introduced the concept of rectangular metric space (RMS), and proved an analog of the Banach contraction principle in such spaces. Since then many fixed point theorems for various contractions on RMS appeared (see [10-14]).

We have the following diagram where arrows stand for inclusions. The inverse inclusions do not hold.

$$\begin{tikzcd}
\textit{metric space} \arrow[r] \arrow[d] & \text{b}-\textit{metric space} \arrow[d] \\
\textit{rectangular metric space} \arrow[r] & \textit{rectangular } \text{b}-\textit{metric space}
\end{tikzcd}$$

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George [1] put forward the following theorem, which is the analog to Banach contraction principle in rectangular b-metric space.

Theorem 1.2[1] Let (X,d) be a complete rectangular b-metric space with coefficient $s>1$ and T be a mapping satisfying:

$$d(Tx,Ty)\leq \lambda d(x,y)$$

for all $x,y\in X$, where $\lambda\in[0,\frac{1}{s}]$. Then T has a unique fixed point.

At the end of this paper, the author gave two open problems as follows:

Problem 1: In Theorem 1.4 can we extend the range of λ to the case $\frac{1}{s}<\lambda<1$.

Problem 2: Prove analogue of Chatterjee contraction, Reich contraction, Ciric contraction and Hardy-Rogers contraction in RbMS.

In [15], the author provided a complete solution to Problem 1, extended the range of λ to the case $\frac{1}{s}<\lambda<1$. Huang HuaPing [16] give a useful lemma, which is the result of the Cauchy sequence in b-metric space. Inspired by the above, we obtain a generalized fixed point result of Reich contraction in b-metric space. In this paper, we present an analogue of Reich contraction in rectangular b-metric space, which is an answer to Problem 2.

In this article, we obtain some Reich type fixed point theorems in generalized metric spaces in section 3. The Theorem 3.1 greatly improved and generalized the previous result from [8]. In fact, this theorem turns out to ba a generalized Reich type fixed point result. Furthermore, Theorem 3.2 shows that in a rectangular b-metric space, the Reich type contraction fixed point theorem is also founded. In addition, we give an application of Theorem 3.1, we verify the existence and uniqueness of solution to Reich type contraction mapping in section 4.

2. Preliminaries

First of all, let's recall some definitions and properties of generalized metric spaces.

Definition 2.1[4] *Let X be a nonempty set and the mapping $d: X\times X\to[0,\infty)$ satisfies:*

$(bM1)$ $d(x,y)=0$ *if and only if* $x=y$ *for all* $x,y\in X$*;*

$(bM2)$ $d(x,y)=d(y,x)$ *for all* $x,y\in X$*;*

$(bM3)$ *there exist a real number* $s\geq 1$ *such that* $d(x,y)\leq s[d(x,z)+d(z,y)]$ *for all* $x,y,z\in X$*.*

Then d is called a b-metric on X and (X,d) is called a b-metric space (in short bMS) with coefficient s.

Definition 2.2[9] *Let X be a nonempty set and the mapping $d: X\times X\to[0,\infty)$ satisfies:*

$(RbM1)$ $d(x,y)=0$ *if and only if* $x=y$ *for all* $x,y\in X$*;*

$(RbM2)$ $d(x,y)=d(y,x)$ *for all* $x,y\in X$*;*

$(RbM3)$ *there exist a real number* $s\geq 1$ *such that* $d(x,y)\leq s[d(x,u)+d(u,v)+d(v,y)]$ *for all* $u,v\in X\backslash\{x,y\}$*.*

Then d is called a rectangular b-metric on X and (X,d) is called a rectangular b-metric space (in short RbMS) with coefficient s.

Definition 2.3[16] *Let (X,d) be a b-metric space or a rectangular b-metric space and $\{x_n\}$ a sequence in X. Then*

(1) $\{x_n\}$ *b-converges to* $x\in X$ *if* $d(x_n,x)\to 0$ *as* $n\to\infty$*;*

(2) $\{x_n\}$ *is a b-Cauchy sequence if* $d(x_m,x_n)\to 0$ *as* $m,n\to\infty$*;*

(3) (X,d) *is a b-complete if every b-Cauchy sequence in* X *is b-convergent.*

Before obtaining the main conclusions, we firstly give an useful lemma, which greatly generalizes and implements the counterpart of the existing literature.

Lemma 2.4[16] Let (X,d) be a b-metric space with coefficient $s \geq 1$ and $T : X \to X$ be a mapping. Suppose that $\{x_n\}$ is a sequence in X induced by $x_{n+1} = Tx_n$ such that

$$d(x_n, x_{n+1}) \leq \lambda d(x_{n-1}, x_n) \tag{2.1}$$

for all $n \in N$, where $\lambda \in [0,1)$ is a constant. Then $\{x_n\}$ is a b-Cauchy sequence.

3. MAIN RESULTS

Firstly, we obtain a Reich contraction fixed point theorem in b-metric space.

Theorem 3.1 *Let* (X,d) *be a complete b-metric space with coefficient* $s \geq 1$ *and* $T : X \to X$ *be a mapping satisfying :*

$$d(Tx,Ty) \leq \lambda d(x,y) + \mu d(x,Tx) + \delta d(y,Ty) \tag{3.1}$$

for all $x, y \in X$*, where* λ, μ, δ *are nonnegative constants with* $\lambda + \mu + \delta < 1$*. Then* T *has a unique fixed point.*

Proof Let $x_0 \in X$ be arbitrary. Then define a sequence $\{x_n\}$ by $x_{n+1} = Tx_n$ for all $n \geq 0$. We shall show that $\{x_n\}$ is $b-$Cauchy sequence.

If $x_n = x_{n+1}$ then x_n is a fixed point of T. So suppose that, for all $n \geq 0$, $x_n \neq x_{n+1}$. Setting $d_n = d(x_n, x_{n+1})$, it follows from (2) that

$$\begin{aligned} d(x_n, x_{n+1}) = d(Tx_{n-1}, Tx_n) &\leq \lambda d(x_{n-1}, x_n) + \mu d(x_{n-1}, Tx_{n-1}) + \delta d(x_n, Tx_n) \\ &= \lambda d(x_{n-1}, x_n) + \mu d(x_{n-1}, x_n) + \delta d(x_n, x_{n+1}) \end{aligned}$$

i.e.

$$d(x_n, x_{n+1}) \leq \frac{\lambda + \mu}{1 - \delta} d(x_{n-1}, x_n)$$

Let $\frac{\lambda+\mu}{1-\delta} = q$, obviously $0 \leq q < 1$, and

$$d_n \leq q d_{n-1}.$$

By Lemma 2.4, now we divide the proof into three cases.

Case 1 $q \in [0, \frac{1}{s})(s > 1)$. By (2.1), we have

$$d(x_n, x_{n+1}) \leq q d(x_{n-1}, x_n) \leq q^2 d(x_{n-2}, x_{n-1}) \leq \cdots \leq q^n d(x_0, x_1)$$

Thus,for any $m > n$, and $n, m \in N$, we have

$$\begin{aligned} d(x_n, x_m) &\leq s[d(x_n, x_{n+1}) + d(x_{n+1}, x_m)] \\ &\leq sd_n + s^2[d(x_{n+1}, x_{n+2}) + d(x_{n+2}, x_m)] \\ &\leq sd_n + s^2 d_{n+1} + s^3 d_{n+2} + \cdots + s^{m-n} d_{m-1} \\ &\leq sq^n d_o + s^2 q^{n+1} d_0 + \cdots + s^{m-n} q^{m-1} d_0 \\ &= sq^n(1 + sq + s^2q^2 + \cdots + s^{m-n-1} q^{m-n-1}) d_0 \\ &\leq sq^n d_0 [\sum_{i=0}^{\infty} (sq)^i] \\ &\leq \frac{sq^n}{1 - sq} d_0 \end{aligned}$$

Now taking limit $n \to \infty$, we get

$$\lim_{n \to \infty} d(x_m, x_n) = 0$$

$\Longrightarrow \{x_n\}$ is b-Cauchy sequence in X.

Case 2 Let $q \in [\frac{1}{s}, 1)$. In this case, we have $q^n \to 0$ as $n \to 0$, so there is $n_0 \in N$, such that $q^{n_0} < \frac{1}{s}$. Thus, by Case 1, we can apply that

$$\{(T^{n_0+n})x_0\}_{n=0}^{\infty} = \{x_{n_0}, x_{n_0+1}, x_{n_0+2}, \dots, x_{n_0+n}, \dots\}$$

is a b-Cauchy sequence. Then

$$\{x_n\}_{n=0}^{\infty} = \{x_0, x_1, x_2, \dots, x_{n_0-1}\} \cup \{x_{n_0}, x_{n_0+1}, x_{n_0+2}, \dots, x_{n_0+n}, \dots\}$$

is a b-Cauchy sequence in X.

Case 3 Let $s = 1$. Similar to the process of Case 1, the claim holds.

Overall, for all $n \in N$, where $0 < q < 1$ is a constant. Then $\{x_n\}$ is a b-Cauchy sequence. So there exists $x^* \in X$, s.t., $\{x_n\}$ converges to x^*.

Now we show that x^* is a fixed point of T. Again, for any $n \in N$ we have

$$d(x_{n+1}, Tx^*) = d(Tx^*, Tx_n) \le \lambda d(x_n, x^*) + \mu d(x^*, Tx^*) + \delta d(x_n, x_{n+1})$$

Where $n \to \infty$, we get

$$d(x^*, Tx^*) \le \lambda d(x^*, x^*) + \mu d(x^*, Tx^*) + \delta d(x^*, x^*) = \mu d(x^*, Tx^*)$$

$\Longrightarrow Tx^* = x^*$, Thus x^* is a fixed point of T. For uniqueness, let y^* be another fixed point of T. Then

$$d(x^*, y^*) = d(Tx^*, Ty^*) \le \lambda d(y^*, x^*) + \mu d(x^*, Tx^*) + \delta d(y^*, Ty^*)] = \lambda d(y^*, x^*)$$

So we have,

$$d(x^*, y^*) \le \lambda d(y^*, x^*) < d(y^*, x^*)$$

a contradiction. Therefore,

$$d(x^*, y^*) = 0$$

i.e.,

$$x^* = y^*$$

Thus fixed point is unique. ■

Remark: Theorem 3.1 generalized the result in [8], relaxed the contraction condition from $\lambda + s(\mu + \delta) \in [0, 1)$ to $\lambda + \mu + \delta \in [0, 1)$.

In what follows, we'll give a Reich type contraction fixed point theorem in rectangular b-metric space.

Theorem 3.2 *let (X, d) be a complete rectangular b-metric space with coefficient $s \ge 1$. T: $X \to X$ be a mapping satisfying :*

$$d(Tx, Ty) \le \lambda d(x, y) + \mu d(x, Tx) + \delta d(y, Ty) \tag{3.2}$$

for all $x, y \in X$, where λ, μ, δ are nonnegative constants with $\lambda + \mu + \delta < 1$. Then T has a unique fixed point.

Proof Let $x_0 \in X$ be arbitrary. Then define a sequence $\{x_n\}$ by $x_{n+1} = Tx_n$ for all $n \in N$. We shall show that $\{x_n\}$ is b-Cauchy sequence.

If $x_n = x_{n+1}$ then x_n is a fixed point of T. So suppose that, for all $n \ge 0$, $x_n \ne x_{n+1}$. Setting $d_n = d(x_n, x_{n+1})$, it follows from (3.2) that

$$\begin{aligned} d(x_n, x_{n+1}) = d(Tx_{n-1}, Tx_n) &\le \lambda d(x_{n-1}, x_n) + \mu d(x_{n-1}, Tx_{n-1}) + \delta d(x_n, Tx_n) \\ &= \lambda d(x_{n-1}, x_n) + \mu d(x_{n-1}, x_n) + \delta d(x_n, x_{n+1}) \end{aligned}$$

i.e.

$$d(x_n, x_{n+1}) \le \frac{\lambda + \mu}{1 - \delta} d(x_{n-1}, x_n)$$

Let $\frac{\lambda+\mu}{1-\delta} = q$, obviously $0 \leq q < 1$, and

$$d_n \leq q d_{n-1}.$$

$0 \leq \frac{\lambda}{q} < 1$, since $\lambda \in [0,1)$ and $\lim_{n\to\infty} x_n = x_0$, there exists a natural number N such that, for all $k \leq N$, $0 \leq s\lambda^k < 1$.

Repeating this process we obtain

$$d_n \leq q^n d_0 \tag{3.3}$$

We can assume that x_0 is not a periodic point of T. If $x_0 = x_n$, then using (3.3), for any $n \geq 2$, we have

$$\begin{array}{rcl} d(x_0, Tx_0) & = & d(x_n, Tx_n) \\ d(x_0, x_1) & = & d(x_n, x_{n+1}) \\ d_0 & = & d_n \\ d_0 & \leq & q^n d_0 \end{array}$$

a contradiction. Therefore, there must be $d_0 = 0, i.e., x_0 = x_1$, and so x_0 is a fixed point of T. Thus we assume that $x_n \neq x_m$ for all distinct $m, n \in N$. Since (X, d) is rectangular b-metric space, from condition $(RbM3)$ we have

$$d(x_m, x_n) \leq s[d(x_m, x_{m+k}) + d(x_{m+k}, x_{n+k}) + d(x_{n+k}, x_n)]$$

First, let us discuss $d(x_n, x_{n+k})$, for all $k \in N$.

$$\begin{array}{rcl} d(x_n, x_{n+k}) & \leq & \lambda d(x_{n-1}, x_{n+k-1}) + \mu d(x_{n-1}, x_n) + \delta d(x_{n+k-1}, x_{n+k}) \\ & \leq & \lambda[\lambda d(x_{n-2}, x_{n+k-2}) + \mu d(x_{n-2}, x_{n-1}) + \delta d(x_{n+k-2}, x_{n+k-1})] + \mu d_{n-1} + \delta d_{n+k-1} \\ & \leq & \lambda^2[\lambda[\lambda d(x_{n-3}, x_{n+k-3}) + \mu d(x_{n-3}, x_{n-2}) + \delta d(x_{n+k-3}, x_{n+k-2})] \\ & & +\lambda\mu d_{n-2} + \mu d_{n-1} + \lambda\delta d_{n+k-2} + \delta d_{n+k-1} \\ & \leq & \cdots \\ & \leq & \lambda^{n-1}[\lambda d(x_0, x_k) + \mu d(x_0, x_1) + \delta d(x_k, x_{k+1})] + \lambda^{n-2}\mu d_1 + \cdots + \lambda\mu d_{n-2} + \mu d_{n-1} \\ & & +\lambda^{n-2}\delta d_k + \cdots + \lambda\delta d_{n+k-2} + \delta d_{n+k-1} \\ & = & \lambda^n d(x_0, x_k) + \lambda_{n-1}\mu d_0 + \lambda^{n-2}\mu d_1 + \cdots + \lambda\mu d_{n-2} + \mu d_{n-1} \\ & & +\lambda_{n-1}\delta d_k + \lambda^{n-2}\delta d_{k+1} + \cdots + \lambda\delta d_{n+k-2} + \delta d_{n+k-1} \\ & \leq & \lambda^n d(x_0, x_k) + \mu d_0(\lambda^{n-1} + \lambda^{n-2}q + \cdots + \lambda q^{n-2} + q^{n-1}) + \\ & + & \delta d_0 q^k(\lambda^{n-1} + \lambda^{n-2}q + \cdots + \lambda q^{n-2} + q^{n-1}) \\ & \leq & \lambda^n d(x_0, x_k) + \dfrac{q^{n-1}[1-(\frac{\lambda}{q})^n]}{1-\frac{\lambda}{q}}\mu d_0 + \dfrac{q^{n-1}[1-(\frac{\lambda}{q})^n]}{1-\frac{\lambda}{q}} q^k\delta d_0 \\ & \leq & \lambda^n d(x_0, x_k) + \dfrac{q^{n-1}}{1-\frac{\lambda}{q}}\mu d_0 + \dfrac{q^{n-1}}{1-\frac{\lambda}{q}} q^k\delta d_0 \end{array}$$

$$\begin{aligned}
d(x_{n+k}, x_{m+k}) &\leq \lambda d(x_{n+k-1}, x_{m+k-1}) + \mu d(x_{n+k-1}, x_{n+k}) + \delta d(x_{m+k-1}, x_{m+k}) \\
&\leq \lambda[\lambda d(x_{n+k-2}, x_{m+k-2}) + \mu d(x_{n+k-2}, x_{n+k-1}) + \delta d(x_{m+k-2}, x_{m+k-1})] \\
&\quad + \mu d_{n+k+1} + \delta d_{m+k-1} \\
&\leq \dots \\
&\leq \lambda^{k-1}[\lambda d(x_n, x_m) + \mu d(x_n, x_{n+1}) + \delta d(x_m, x_{m+1})] + \lambda^{k-2}\mu d_{n+1} + \lambda^{k-2}\delta d_{m+1} + \dots \\
&\quad + \lambda\mu d_{n+k-2} + \lambda\delta d_{m+k-2} + \mu d_{n+k-1} + \delta d_{m+k-1} \\
&= \lambda^k d(x_n, x_m) + \lambda^{k-1}\mu d_n + \lambda^{k-2}\mu d_{n+1} + \dots \\
&\quad + \lambda^{k-2}\mu d_{n+1} + \mu d_{n+k-1} + \lambda^{k-1}\delta d_m + \lambda^{k-2}\delta d_{m+1} + \dots + \lambda^{k-2}\delta d_{m+1} + \mu d_{m+k-1} \\
&\leq \lambda^k d(x_n, x_m) + \frac{q^{k-1}[1 - (\frac{\lambda}{q})^k]}{1 - \frac{\lambda}{q}} q^n \mu d_0 + \frac{q^{k-1}[1 - (\frac{\lambda}{q})^k]}{1 - \frac{\lambda}{q}} q^m \delta d_0 \\
&\leq \lambda^k d(x_n, x_m) + \frac{q^{k-1}}{1 - \frac{\lambda}{q}} q^n \mu d_0 + \frac{q^{k-1}}{1 - \frac{\lambda}{q}} q^m \delta d_0
\end{aligned}$$

In summary

$$\begin{aligned}
d(x_m, x_n) &\leq s[d(x_m, x_{m+k}) + d(x_{m+k}, x_{n+k}) + d(x_{n+k}, x_n)] \\
&\leq s[\lambda^m d(x_0, x_k) + \frac{q^{m-1}}{1 - \frac{\lambda}{q}} \mu d_0 + \frac{q^{m-1}}{1 - \frac{\lambda}{q}} q^k \delta d_0 \\
&\quad + \lambda^k d(x_n, x_m) + \frac{q^{k-1}}{1 - \frac{\lambda}{q}} q^n \mu d_0 + \frac{q^{k-1}}{1 - \frac{\lambda}{q}} q^m \delta d_0 \\
&\quad + \lambda^n d(x_0, x_k) + \frac{q^{n-1}}{1 - \frac{\lambda}{q}} \mu d_0 + \frac{q^{n-1}}{1 - \frac{\lambda}{q}} q^k \delta d_0]
\end{aligned}$$

$$(1 - s\lambda^k) d(x_m, x_n) \leq s[(\lambda^m + \lambda^n) d(x_0, x_k) + \frac{q^{m-1} + q^{n-1} + q^{k+n-1}}{1 - \frac{\lambda}{q}} \mu d_0 + \frac{q^{m-1} + q^{n-1} + q^{k+m-1}}{1 - \frac{\lambda}{q}} q^k \delta d_0]$$

From this, together with before, we obtain

$$d(x_m, x_n) \leq s \frac{\lambda^m + \lambda^n}{1 - s\lambda^k} d(x_0, x_k) + s \frac{q^{m-1} + q^{n-1} + q^{k+n-1}}{(1 - \frac{\lambda}{q})(1 - s\lambda^k)} \mu d_0 + s \frac{q^{m-1} + q^{n-1} + q^{k+m-1}}{(1 - \frac{\lambda}{q})(1 - s\lambda^k)} q^k \delta d_0$$

Since $\lambda \in [0, 1)$, thus there exist $k \in N$, s.t. $s\lambda^k < 1$, so we can apply that $\{x_n\}$ is a b-Cauchy sequence in X. By completeness of (X, d) there exist $x^* \in X$, such that

$$\lim_{n \to \infty} x_n = x^*$$

Now we show that x^* is a fixed point of T. Again, for any $n \in N$ we have

$$\begin{aligned}
d(x_{n+1}, Tx^*) &= d(Tx^*, Tx_n) \\
&\leq \lambda d(x_n, x^*) + \mu d(x^*, Tx^*) + \delta d(x_n, x_{n+1})
\end{aligned}$$

Where $n \to \infty$, we get

$$d(x^*, Tx^*) \leq \lambda d(x^*, x^*) + \mu d(x^*, Tx^*) + \delta d(x^*, x^*) = \mu d(x^*, Tx^*)$$

$\longrightarrow Tx^* = x^*$, Thus x^* is a fixed point of T. For uniqueness, let y^* be another fixed point of T. Then

$$d(x^*, y^*) = d(Tx^*, Ty^*) \leq \lambda d(y^*, x^*) + \mu d(x^*, Tx^*) + \delta d(y^*, Ty^*)] = \lambda d(y^*, x^*)$$

So we have,

$$d(x^*, y^*) \leq \lambda d(y^*, x^*) < d(y^*, x^*).$$

a contradiction. Therefore, we must have $d(x^*, y^*) = 0$, i.e., $x^* = y^*$. Thus fixed point is unique. ■

Remark Theorem 3.2 is an answer of the open problem 2, it proves an analogue of Reich contraction in rectangular b-metric space.

4. Examples

Example 4.1 *Let* $p \in (0,1)$

$$X = L^p[0,1] := \{x : [0,1] \to R| \int_0^1 |x(t)|^p dt < \infty\}.$$

For $x, y \in X$*, let*

$$d(x,y) = (\int_0^1 |x_n - y_n|^p dt)^{\frac{1}{p}}$$

Then (X, d) is a complete b-metric space with coefficient $s = 2^{\frac{1}{p}} > 1$. Defined $T : X \to X$ as

$$Tx = \frac{1}{3}(x + sinx)$$

We assert that for $\forall x, y \in X$,

$$d(Tx, Ty) \leq \frac{2}{3}d(x,y) + \frac{1}{8}d(x, Tx) + \frac{1}{9}d(y, Ty)$$

Clearly, T satisfies the condition of Theorem 3.1, and has a unique fixed point of $x = 0$.

Example 4.2 *Let* $X = \{0, 1, \frac{1}{2}, \frac{1}{3}, \dots, \frac{1}{n}, \dots\}, n \in N^+$*, let*

$$d(x,y) = \begin{cases} 0 & x = y \\ 2 & x \neq y \in \{0,1\} \\ |x - y| & otherwise \end{cases}$$

Then $(X, d)$is a b-metric space with coefficient $s = 2 > 1$, and let $T : X \longrightarrow X$ be defined by

$$Tx = \frac{x}{2}$$

We have

$$d(Tx, Ty) \leq \frac{1}{2}d(x,y) + \frac{1}{4}d(x, Tx) + \frac{1}{5}d(y, Ty)$$

T satisfies the condition of Theorem 3.1 and has a unique point of $x = 0$.

5. Conclusion

Inspired by Reny George, using iterative method, in this paper we discuss Reich contraction fixed point theorems in b-metric space and $RbMS$. We obtained a generalized theorem that satisfies the weaker compression condition. And the latter theorem solved an open problem of [1].

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References

[1] R. George, S. Radenovic, K. P. Reshma, S. Shukla, *Rectangular b-metric spaces and contraction principle*, J. Nonlinear Sci. Appl. **8** (2015) 1005-1013.

[2] S. Czerwik, *Contraction mappings in b-metric Spaces*, Acta. Math. Inf. Univ. Ostrav. **1** (1993) 5-11.

[3] S. Czerwik, *Nonlinear set-valued contraction mappings in b-metric spaces*, Atti. Sem. Mat. Fis. Univ. Modena **46** (1998) 263-276.

[4] I. A. Bakhtin, *The contraction mapping principle in quasimetric spaces, Funct. Anal.*, Unianowsk Gos. Ped. Inst. **30** (1989) 26-37.

[5] M. Boriceanu, *Strict fixed point theorems for multivalued operators in b-metric spaces*, International J. Modern Math. **4** **3** (2009) 285-301.

[6] N. V. Van, VTL. Hang, *On relaxations of contraction constants and Caristi's theorem in b-metric spaces*, J. Fixed Point Theory Appl. **18** (2016), 267-284.

[7] M. Bota, A. Molnar, C. Varga, *On ekeland's variational principle in b-metric spaces*, Fixed Point Theory **12** (2011), no. 2, 21-28.

[8] K. M. Pankaj,S. Shweta, S. K. Banerjee, *Some fixed point theorems in b-metric space*, Turkish J. Anal. Number Theory **2** **1** (2014) 19-22.

[9] A. Branciari, *Fixed point theorem of Banach-Caccippoli type on a class of generalised metric spaces*, Publ. Math. Debrecen **57** (2000) 31-37.

[10] P. Das, *A fixed point theorem in generalised metric spaces*, Soochow J. Math. **33** (2007) 33-39.

[11] P. Das, *A fixed point theorem on a class of generalised metric spaces*, Korean J. Math. Sci. **9** (2002) 29-33.

[12] A. Azam,M. Arshad, *Kannan Fixed Point Theorems on generalised metric spaces*, J. Nonlinear Sci. Appl. **1** (2008) 45-48.

[13] A. Azam, M. Arshad,I. Beg, *Banach contraction principle on cone rectangular metric spaces*, Appl. Anal. Discrete Math. **3** (2009) 236-241.

[14] P. Das, *A fixed point theorem in generalised metric spaces*. Soochow J. Math. **33** (2007) 33-39.

[15] Z. D. Mitrovic, *On an open problem in rectangular b-metric space*, J. Anal.**25** (2017) 135-137.

[16] H. Huang, G. Deng and R. Stojan, *Fixed point theorems in b-metric spaces with applications to differential equations*, J. Fixed Point Theory Appl. **20** **1** (2018) 52.

[17] H. Afshari, H. Aydi, E.Karapinar, *Existence of Fixed Points of Set-Valued Mappings in b-Metric Spaces*, East Asian mathematical journal , volume 32, issue 3, 2016, Pages 319 332, DOI : 10.7858/eamj.2016.024.

[18] U.Aksoy, E. Karapinar and I. M. Erhan, *Fixed points of generalized alpha-admissible contractions on b-metric spaces with an application to boundary value problems*, Journal of Nonlinear and Convex Analysis Volume 17, Number 6 (2016) 1095-1108.

[19] H. Alsulami, S.Gulyaz, E. Karapinar, I. Erhan, *An Ulam stability result on quasi-b-metric-like spaces*, Open Mathematics, Volume 14, Issue 1 (Jan 2016) DOI 10.1515/math-2016-0097.

[20] H. Aydi,M-F.Bota, E. Karapinar, S.Moradi, *A Common Fixed Point For Weak -Phi-Contractions on b-Metric Spaces*, Fixed Point Theory **13** **2** (2012) 337-346.

[21] M.-F. Bota, E. Karapinar and O. Mlesnite, *Ulam-Hyers stability results for fixed point problems via alpha-psi-contractive mapping in b-metric space*, Abstract and Applied Analysis (2013) Article Id: 825293.

[22] M-F.Bota, E. Karapinar, *A note on "Some results on multi-valued weakly Jungck mappings in b-metric space*, Cent. Eur. J. Math. **11** **9** (2013) 1711–1712, DOI: 10.2478/s11533-013-0272-2.

[23] M. Bota, C. Chifu, E. Karapinar, *Fixed point theorems for generalized (alpha-psi)-Ciric-type contractive multivalued operators in b-metric spaces*, J. Nonlinear Sci. Appl. **9** (2016) 1165-1177.

[24] Selma Gulyaz Ozyurt, *On some alpha-admissible contraction mappings on Branciari b-metric spaces*, Advances in the Theory of Nonlinear Analysis and its Applications **1** **1** (2017) 1-13.

[25] E. Karapinar, H.Piri and H.H. AlSulami, *Fixed Points of Generalized F-Suzuki Type Contraction in Complete b-Metric Spaces* , Discrete Dynamics in Nature and Society (2015) Article ID 969726, 8 pages.

[26] M.A. Kutbi , E. Karapinar, J. Ahmed, A. Azam, *Some fixed point results for multi-valued mappings in b-metric spaces* , Journal of Inequalities and Applications (2014), 2014:126.

[27] M. Berzig, E. Karapinar, A.Roldan, *Some Fixed Point Theorems In Branciari Metric Spaces*, Mathematica Slovaca **67 5** (2017) 1-14.

[28] M.Arshad, E. Ameer, E. Karapinar, *Generalized contractions with triangular alpha-orbital admissible mapping on Branciari metric spaces*, Journal of Inequalities and Applications (2016) 2016:63 .

[29] H. Aydi, E.Karapinar, D. Zhang, *A note on generalized admissible-Meir-Keeler-contractions in the context of generalized metric spaces*, Results in Mathematics **71 1** (2017) 73-92.

[30] E. Karapinar, *Some fixed points results on Branciari metric spaces via implicit functions*, Carpathian J. Math. **31 3** (2015) 339 - 348.

[31] M.Jleli, E. Karapinar, and B.Samet, *Further generalizations of the Banach contraction principle*, Journal of Inequalities and Applications (2014) 2014:439.

[32] M.Asadi, E. Karapinar and A. Kumar, $\alpha - \psi$*-Geraghty contractions on generalized metric spaces*, Journal of Inequalities and Applications (2014) 2014:423.

[33] C.-M. Chen, E. Karapinar and I.-J. Lin, *Periodic points of weaker Meir-Keeler contractive mappings on generalized quasi-metric spaces*, Abstract and Applied Analysis (2014) Article No: 490450.

[34] E. Karapinar, H.Aydi and B.Samet, *Fixed points for generalized (alpha,psi)-contractions on generalized metric spaces*, Journal of Inequalities and Applications (2014) 2014:229.

[35] M. A. Alghamdi, C.-M. Chen and E.Karapinar, *A generalized weaker (alpha-phi-varphi)-contractive mappings and related fixed point results in complete generalized metric spaces*, Abstract and Applied Analysis (2014) Article Id:985080.

[36] N.Bilgili, E. Karapinar and D.Turkoglu, *A note on common fixed points for (psi,alpha,beta)-weakly contractive mappings in generalized metric spaces*, Fixed Point Theory and Applications (2013) 2013:287.

[37] I. M. Erhan, E. Karapinar and T. Sekulic, *Fixed points of (psi,phi) contractions on rectangular metric spaces*, Fixed Point Theory Appl. (2012) 2012:138.

[38] M. Arshad,J. Ahmad, E. Karapinar, *Some Common Fixed Point Results in Rectangular Metric Spaces*, International Journal of Analysis, Volume 2013 (2013), Article ID 307234, 7 pages.

[39] E. Karapinar and A. Pitea, *On alpha-psi-Geraghty contraction type mappings on quasi-Branciari metric spaces*, Journal of Nonlinear and Convex Analysis **17 7** (2016) 1291-1301.

1. School of Science, Wuhan University of Science and Technology,, Wuhan 430065, P.R. China., 2. Hubei Province Key Laboratory of Systems Science in Metallurgical Process,, Wuhan 430065, P.R. China.

E-mail address: `E-mail:1261498279@qq.com`

E-mail address: `Corresponding author,E-mail:yqfeng6@126.com`