

S-NORMALITY

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ABSTRACT. A topological space X is called S -normal if there exists a normal space Y and a bijective function $f : X \rightarrow Y$ such that the restriction $f|_A : A \rightarrow f(A)$ is a homeomorphism for each separable subspace $A \subseteq X$. We will investigate this property and produce some examples to illustrate the relation between S -normality and other weaker kinds of normality.

Arhangel'skiĭ introduced on 2012 a new weaker version of normality, called C -normality [7]. A topological space X is called C -normal if there exists a normal space Y and a bijective function $f : X \rightarrow Y$ such that the restriction $f|_C : C \rightarrow f(C)$ is a homeomorphism for each compact subspace $C \subseteq X$. We used the idea of this definition to introduce another weaker version of normality and called it S -normality. We investigate in this paper the S -normal property. We prove that normality implies S -normality but the converse is not true in general. We present some examples to show relationships between S -normality and other weaker versions of normality such as C -normality, L -normality, CC -normal, mild normality. Throughout this paper, we denote an ordered pair by $\langle x, y \rangle$, the set of positive integers by \mathbb{N} , the rationals by \mathbb{Q} , and the set of real numbers by \mathbb{R} . A T_4 space is a T_1 normal space and a Tychonoff space is a T_1 completely regular space. We do not assume T_2 in the definition of compactness and countable compactness. We do not assume regularity in the definition of Lindelöfness. For a subset A of a space X , $\text{int}A$ and \bar{A} denote the interior and the closure of A , respectively. An ordinal γ is the set of all ordinals α such that $\alpha < \gamma$. The first infinite ordinal is ω_0 , the first uncountable ordinal is ω_1 , and the successor cardinal of ω_1 is ω_2 .

Definition 1. A topological space X is called S -normal if there exists a normal space Y and a bijective function $f : X \rightarrow Y$ such that the restriction $f|_A : A \rightarrow f(A)$ is a homeomorphism for each separable subspace $A \subseteq X$.

Obviously, any normal space is S -normal, just take $X = Y$ and the identity function. Here is an example of a Tychonoff S -normal space which is not normal.

Example 2. We modify the Dieudonné Plank [11] to define a new topological space. Let

$$X = ((\omega_2 + 1) \times (\omega_0 + 1)) \setminus \{\langle \omega_2, \omega_0 \rangle\}.$$

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Write $X = A \cup B \cup N$, where $A = \{\langle \omega_2, n \rangle : n < \omega_0\}$, $B = \{\langle \alpha, \omega_0 \rangle : \alpha < \omega_2\}$, and $N = \{\langle \alpha, n \rangle : \alpha < \omega_2 \text{ and } n < \omega_0\}$. The topology τ on X is generated by the following neighborhood system: For each $\langle \alpha, n \rangle \in N$, let $\mathcal{B}(\langle \alpha, n \rangle) = \{\{\langle \alpha, n \rangle\}\}$. For each $\langle \omega_2, n \rangle \in A$, let $\mathcal{B}(\langle \omega_2, n \rangle) = \{V_\alpha(n) = (\alpha, \omega_2] \times \{n\} : \alpha < \omega_2\}$. For each $\langle \alpha, \omega_0 \rangle \in B$, let $\mathcal{B}(\langle \alpha, \omega_0 \rangle) = \{V_n(\alpha) = \{\alpha\} \times (n, \omega_0] : n < \omega_0\}$, see [3, 4]. Then X is a Tychonoff space, because it is T_1 zero-dimensional, which is not normal because A and B are disjoint closed sets that cannot be separated by disjoint open sets. Now, a subspace $C \subseteq X$ is separable if and only if C is countable. To see this, suppose that C is uncountable. Then either $C \cap N$ is uncountable or $C \cap B$ is uncountable. If $C \cap N$ is uncountable, pick any countable subset D of C and pick $x \in (C \cap N) \setminus D$. Then $\{x\}$ is a non-empty open set disjoint from D , hence D is not dense, thus C is not separable which is a contradiction. If $C \cap B$ is uncountable but $C \cap N$ is countable. Let D be any countable subset of C . Then there exists a $\gamma < \omega_2$ such that $\langle \gamma, \omega_0 \rangle \in C \cap B$ and $V_0(\gamma) \cap (D \cup (C \cap N)) = \emptyset$. Hence D is not dense, thus C is not separable which is a contradiction.

Now, define $Y = X = A \cup B \cup N$. Generate a topology τ' on Y by the following neighborhood system: Elements of $B \cup N$ have the same local base as in X . For each $\langle \omega_2, n \rangle \in A$, let $\mathcal{B}(\langle \omega_2, n \rangle) = \{\{\langle \omega_2, n \rangle\}\}$. Then Y is T_4 space because it is Hausdorff paracompact. Consider the identity function $id : X \rightarrow Y$. So, id is a bijective function. Let $C \subset X$ be any separable subspace. Then $id \upharpoonright_C : C \rightarrow id(C) = C$ is a bijective. Let $\langle a, b \rangle$ be any element in C . If $\langle a, b \rangle \in N$, then $\{\langle a, b \rangle\}$ which is open in C as a subspace of X and Y will give that $id \upharpoonright_C$ is continuous. If $\langle a, b \rangle \in B$ and W is any basic open set of $\langle a, b \rangle$ in C as a subspace of Y , then W is also a basic open set of $\langle a, b \rangle$ in C as a subspace of X , hence $id \upharpoonright_C$ is continuous. If $\langle a, b \rangle \in A$, then the smallest open neighborhood of $\langle a, b \rangle$ in C as a subspace of Y is $\{\langle a, b \rangle\}$. Since C is separable in X , then it is countable. Write $(\omega_2 \times \{b\}) \cap C = \{\langle \alpha_k, b \rangle : k \in K\}$ where K is countable. Pick $\beta < \omega_2$ such that $\alpha_k < \beta$ for each $k \in K$. Then $V_\beta(b)$ is a basic open set of $\langle a, b \rangle$ in X , hence $V_\beta(b) \cap C = \{\langle a, b \rangle\}$ is an open neighborhood of $\langle a, b \rangle$ in C as a subspace of X . Thus $id \upharpoonright_C$ is continuous. From the three cases, we conclude $id \upharpoonright_C$ is continuous. Since the topology on X is coarser than the topology on Y , the inverse function of $id \upharpoonright_C$ is also continuous. We conclude that $id \upharpoonright_C$ is a homeomorphism. Therefore, the modified Dieudonné Plank X is S -normal. ■

It is clear from the definition that any separable S -normal space must be normal. Thus the Niemytzki Plane [11], the Sorgenfrey line square [11], and the rational sequence topology on \mathbb{R} [11], are all Tychonoff spaces which are not S -normal. If X is countable, then X may not be S -normal, for example, (\mathbb{Q}, τ_p) , where τ_p is the particular point topology and $p \in \mathbb{Q}$ [11].

Recall that a topological space X is called C -normal if there exists a normal space Y and a bijective function $f : X \rightarrow Y$ such that the restriction $f \upharpoonright_A : A \rightarrow f(A)$ is a homeomorphism for each compact subspace $A \subseteq X$ [7]. Here is an example of a Tychonoff C -normal space which is not S -normal.

Example 3. We choose a suitable Mrówka space. Recall that two countably infinite sets are said to be *almost disjoint* [1] if their intersection is finite. Call a subfamily of $[\omega_0]^{\omega_0} = \{A \subseteq \omega_0 : A \text{ is infinite}\}$ a *mad family* [1] on ω_0 if it is a maximal (with respect to inclusion) pairwise almost disjoint subfamily. Let \mathcal{A} be a pairwise almost disjoint subfamily of $[\omega_0]^{\omega_0}$. The *Mrówka space* $\Psi(\mathcal{A})$ is defined as follows: The underlying set is $\omega_0 \cup \mathcal{A}$, each point of ω_0 is isolated, and a basic open neighborhood of $W \in \mathcal{A}$ has the form $\{W\} \cup (W \setminus F)$, with $F \in [\omega_0]^{<\omega_0} = \{B \subseteq \omega_0 : B \text{ is finite}\}$.

It is well known that there exists an almost disjoint family $\mathcal{A} \subset [\omega_0]^{\omega_0}$ such that $|\mathcal{A}| > \omega_0$ and the Mrówka space $\Psi(\mathcal{A})$ is a Tychonoff, separable, first countable, and locally compact space which is neither countably compact nor normal. And \mathcal{A} is a *mad* family if and only if $\Psi(\mathcal{A})$ is pseudocompact [8]. Since $\Psi(\mathcal{A})$ is T_2 locally compact, it is C -normal, see [7]. It is not S -normal because it is separable non-normal space. ■

Here is another simpler example of a Tychonoff separable C -normal space which is not S -normal.

Example 4. Consider the Sorgenfrey line square [11] which is a Tychonoff separable non-normal space, hence cannot be S -normal. Since it is submetrizable, it is C -normal [7]. ■

Here is an example of an S -normal space which is not C -normal.

Example 5. Let $i = \sqrt{-1}$ and $j = \sqrt{-2}$. Let $X = \mathbb{R} \cup \{i, j\}$. For each $x \in \mathbb{R}$, let $\mathcal{B}(x) = \{\{x\}\}$. Let $\mathcal{B}(i) = \{\{i\} \cup (\mathbb{R} \setminus E) : E \subset \mathbb{R} \text{ is finite}\}$. Let $\mathcal{B}(j) = \{\{j\} \cup (\mathbb{R} \setminus E) : E \subset \mathbb{R} \text{ is countable}\}$. Let τ denote the unique topology on X which has $\{\mathcal{B}(x) : x \in X\}$ as its neighborhood system. Then (X, τ) is a compact space which is neither Hausdorff nor normal, hence cannot be C -normal. Now, a subspace C of X is separable if and only if C is countable. We show that (X, τ) is S -normal. Let $Y = X$. For $y \in \mathbb{R} \cup \{i\}$, let $\mathcal{B}(y)$ be the same as in (X, τ) . Let $\mathcal{B}(j) = \{\{j\}\}$. Let τ' denote the unique topology on Y which has $\{\mathcal{B}(y) : y \in Y\}$ as its neighborhood system. Then (Y, τ') is normal being T_2 compact. Let $f : X \rightarrow Y$ be the identity function. Let C be any separable subspace of X . Then C is countable. Pick any $z \in C$. If $z \in \mathbb{R}$, then the smallest open neighborhood of z in C as a subspace of Y is $\{z\}$ which is also open in C as a subspace of X , hence $f \upharpoonright_C$ is continuous at z . If $z = i$, Then a basic open neighborhood of i in C as a subspace of Y is of the form $U \cap C$, where $U = \{i\} \cup (\mathbb{R} \setminus E)$ and $E \subset \mathbb{R}$ is finite, which is also open in C as a subspace of X . Thus $f \upharpoonright_C$ is continuous at $z = i$. If $z = j$, then the smallest open neighborhood of j in C as a subspace of Y is $\{j\}$. Since C is countable, write $C = B \cup \{j\}$ where $B \subset \mathbb{R}$ is countable. Consider $U = \{j\} \cup (\mathbb{R} \setminus B)$, then U is an open neighborhood of j in X such that $U \cap C = \{j\}$ which means that $\{j\}$ is also open in C as a subspace of X , hence $f \upharpoonright_C$ is continuous at $z = j$. Thus $f \upharpoonright_C$ is continuous. Since the topology τ is coarser than τ' , then f^{-1} is continuous. So, $f \upharpoonright_C : C \rightarrow f(C)$ is a homeomorphism. Therefore, (X, τ) is S -normal. ■

Recall that a topological space X is called *L-normal* if there exists a normal space Y and a bijective function $f : X \rightarrow Y$ such that the restriction $f \upharpoonright_A : A \rightarrow f(A)$ is a homeomorphism for each Lindelöf subspace $A \subseteq X$ [4]. $(\mathbb{R}, \mathcal{CC})$, where \mathcal{CC} is the countable complement topology [11], is S -normal but not L -normal. It is not L -normal because it is Lindelöf non-normal space. It is S -normal because the only separable subspace of $(\mathbb{R}, \mathcal{CC})$ are the countable subspaces and since any countable subspace of $(\mathbb{R}, \mathcal{CC})$ is discrete, then $Y = \mathbb{R}$ with the discrete topology and the identity function on \mathbb{R} will show the S -normality of $(\mathbb{R}, \mathcal{CC})$. We still do not know an example of an L -normal space which is not S -normal.

Recall that a topological space X is called *CC-normal* if there exists a normal space Y and a bijective function $f : X \rightarrow Y$ such that the restriction $f \upharpoonright_A : A \rightarrow f(A)$ is a homeomorphism for each countably compact subspace $A \subseteq X$ [3]. $\Psi(\mathcal{A})$

is CC -normal [3], but not S -normal, see Example 3 above. We need the following theorem to give an example of an S -normal space which is not CC -normal.

Theorem 6. *If X is an L -normal space such that each separable subspace is contained in a Lindelöf subspace, then X is S -normal.*

Proof. Let X be any L -normal space such that if A is any separable subspace of X , then there exists a Lindelöf subspace B such that $A \subseteq B$. Let Y be a normal space and $f : X \rightarrow Y$ be a bijective function such that $f \upharpoonright_C : C \rightarrow f(C)$ is a homeomorphism for each Lindelöf subspace C of X . Now, let A be any separable subspace of X . Pick a Lindelöf subspace B of X such that $A \subseteq B$. Then $f \upharpoonright_B : B \rightarrow f(B)$ is a homeomorphism, hence $f \upharpoonright_A : A \rightarrow f(A)$ is a homeomorphism as $(f \upharpoonright_B) \upharpoonright_A = f \upharpoonright_A$. \square

By similar argument, we obtain the following corollary.

Corollary 7. .

- (a) *If X is C -normal and any separable subspace of X is contained in a compact subspace of X , then X is S -normal.*
- (b) *If X is CC -normal and any separable subspace of X is contained in a countably compact subspace of X , then X is S -normal.*
- (c) *If X is S -normal and any Lindelöf subspace of X is contained in a separable subspace of X , then X is L -normal.*
- (d) *If X is S -normal and any compact subspace of X is contained in a separable subspace of X , then X is C -normal.*
- (e) *If X is S -normal and any countably compact subspace of X is contained in a separable subspace of X , then X is CC -normal.*

Here is an example of an S -normal space which is not CC -normal.

Example 8. Consider $\omega_1 \times (\omega_1 + 1)$ which is L -normal, see [4], but not CC -normal because it is countably compact but not normal.

Claim: A subspace C of $\omega_1 \times (\omega_1 + 1)$ is separable if and only if C is countable.

Proof of Claim: Assume that C is a separable subspace of $\omega_1 \times (\omega_1 + 1)$. Suppose that C is uncountable. Let D be an arbitrary countable subset of C . Let $\eta = \sup\{\mu, \xi : \langle \mu, \xi \rangle \in D \cap (\omega_1 \times \omega_1)\}$. Let $B = \{\langle \alpha, \omega_1 \rangle : \alpha < \omega_1\}$. We have just two cases, either $C \cap B$ is uncountable or $C \cap B$ is countable.

Case 1: $C \cap B$ is uncountable. Let $K = \{\alpha < \omega_1 : \langle \alpha, \omega_1 \rangle \in D\}$. Then K is countable. Thus there exists a $\gamma < \omega_1$ such that $\zeta = (\sup K) + 1 < \gamma$ and $\langle \gamma, \omega_1 \rangle \in C$. Then $(\zeta, \gamma] \times (\eta, \omega_1]$ is an open neighborhood of $\langle \gamma, \omega_1 \rangle$ disjoint from D . Hence D is not dense, thus C is not separable which is a contradiction.

Case 2: $C \cap B$ is countable. Then $C \cap (\omega_1 \times \omega_1)$ must be uncountable. So, there exist $\alpha, \beta < \omega_1$ with $\eta < \alpha, \beta$ and $\langle \alpha, \beta \rangle \in C \cap (\omega_1 \times \omega_1)$. Then $(\eta, \alpha] \times (\eta, \beta]$ is an open neighborhood of $\langle \alpha, \beta \rangle$ disjoint from D . Hence D is not dense, thus C is not separable which is a contradiction.

We conclude that any separable subspace of $\omega_1 \times (\omega_1 + 1)$ is Lindelöf. Thus, by Theorem 6, we get that $\omega_1 \times (\omega_1 + 1)$ is S -normal. \blacksquare

Theorem 9. *S -normality is a topological property.*

Proof. Let X be an S -normal space and $X \cong Z$. Let Y be a normal space and $f : X \rightarrow Y$ be a bijective function such that $f \upharpoonright_C : C \rightarrow f(C)$ is a homeomorphism

for each separable subspace C of X . Let $g : Z \rightarrow X$ be a homeomorphism. Then $f \circ g : Z \rightarrow Y$ satisfies all requirements. \square

Theorem 10. *S -normality is an additive property.*

Proof. Let X_α be an S -normal space for each $\alpha \in \Lambda$. We show that their sum $\bigoplus_{\alpha \in \Lambda} X_\alpha$ is S -normal. For each $\alpha \in \Lambda$, pick a normal space Y_α and a bijective function $f_\alpha : X_\alpha \rightarrow Y_\alpha$ such that $f_{\alpha|_{C_\alpha}} : C_\alpha \rightarrow f_\alpha(C_\alpha)$ is a homeomorphism for each separable subspace C_α of X_α . Since Y_α is normal for each $\alpha \in \Lambda$, the sum $\bigoplus_{\alpha \in \Lambda} Y_\alpha$ is normal [2, 2.2.7]. Consider the function sum [2, 2.2.E], $\bigoplus_{\alpha \in \Lambda} f_\alpha : \bigoplus_{\alpha \in \Lambda} X_\alpha \rightarrow \bigoplus_{\alpha \in \Lambda} Y_\alpha$ defined by $\bigoplus_{\alpha \in \Lambda} f_\alpha(x) = f_\beta(x)$ if $x \in X_\beta, \beta \in \Lambda$. Now, a subspace $C \subseteq \bigoplus_{\alpha \in \Lambda} X_\alpha$ is separable if and only if the set $\Lambda_0 = \{\alpha \in \Lambda : C \cap X_\alpha \neq \emptyset\}$ is countable and $C \cap X_\alpha$ is separable in X_α for each $\alpha \in \Lambda_0$. If $C \subseteq \bigoplus_{\alpha \in \Lambda} X_\alpha$ is separable, then $(\bigoplus_{\alpha \in \Lambda} f_\alpha)|_C$ is a homeomorphism because $f_{\alpha|_{C \cap X_\alpha}}$ is a homeomorphism for each $\alpha \in \Lambda_0$. \square

S -normality is not multiplicative because, for example, the Sorgenfrey line is T_4 but its square is a Tychonoff separable space which is not S -normal because it is not normal. Also, S -normality is not hereditary. Take any compactification of the Sorgenfrey line square. We still do not know if S -normality is hereditary with respect to closed subspaces.

A function witnesses the S -normality of a space X may not be continuous, see the identity function in Example 5. But it will be under some conditions. Recall that a space X is of *countable tightness* if for each subset A of X and each $x \in X$ with $x \in \bar{A}$ there exists a countable subset $B \subseteq A$ such that $x \in \bar{B}$ [2]. For a set A , we let $[A]^{\leq \omega_0} = \{B \subseteq A : B \text{ is countable}\}$.

Theorem 11. *If X is S -normal and of countable tightness and $f : X \rightarrow Y$ witnesses the S -normality of X . Then f is continuous.*

Proof. Assume that X is S -normal and of countable tightness. Let $f : X \rightarrow Y$ be any witness of the S -normality of X . Let $A \subseteq X$ be arbitrary. We have

$$f(\bar{A}) = f\left(\bigcup_{B \in [A]^{\leq \omega_0}} \bar{B}\right) = \bigcup_{B \in [A]^{\leq \omega_0}} f(\bar{B}) \subseteq \bigcup_{B \in [A]^{\leq \omega_0}} \overline{f(B)} \subseteq \overline{f(A)}.$$

Therefore, f is continuous \square

Since any first countable space is Fréchet [2, 1.6.14], any Fréchet space is sequential [2, 1.6.14], and any sequential space is of countable tightness [2, 1.7.13(c)], we conclude the following corollary.

Corollary 12. .

- (1) *If X is S -normal first countable and $f : X \rightarrow Y$ witnesses the S -normality of X . Then f is continuous.*
- (2) *If X is S -normal Fréchet and $f : X \rightarrow Y$ witnesses the S -normality of X . Then f is continuous.*
- (3) *If X is S -normal sequential and $f : X \rightarrow Y$ witnesses the S -normality of X . Then f is continuous.*

In general, S -normality is not invariant under quotient mapping. Here is an example.

Example 13. Let $i = \sqrt{-1}$ and $Y = (\mathbb{R} \setminus \mathbb{N}) \cup \{i\}$. Define $f : \mathbb{R} \rightarrow Y$ by

$$f(x) = \begin{cases} x & ; \text{if } x \in \mathbb{R} \setminus \mathbb{N} \\ i & ; \text{if } x \in \mathbb{N} \end{cases}$$

Now consider \mathbb{R} with its usual metric topology \mathcal{U} and define a topology τ on Y by $\tau = \{U \subseteq Y : f^{-1}(U) \in \mathcal{U}\}$. Then τ is a topology on Y and $f : (\mathbb{R}, \mathcal{U}) \rightarrow (Y, \tau)$ is a quotient map. $(\mathbb{R}, \mathcal{U})$ is S -normal being metrizable. Now, (Y, τ) separable because $\mathbb{Q} \setminus \mathbb{N}$ is dense, and T_1 as any singleton is closed. But (Y, τ) is not regular as $E = \{n + \frac{1}{n+1} : n \in \mathbb{N}, n \geq 2\}$ is closed and cannot be separated from i by two disjoint open sets. Thus (Y, τ) is not normal and hence cannot be S -normal. ■

Observe that the function in Example 13 is also closed. Thus S -normality is not closed invariant.

Recall that a subset A of a space X is called *closed domain* [2], called also *regularly closed*, κ -*closed*, if $A = \overline{\text{int}A}$. A space X is called *mildly normal* [9], called also κ -*normal* [10], if for any two disjoint closed domains A and B of X there exist two disjoint open sets U and V of X such that $A \subseteq U$ and $B \subseteq V$, see also [5] and [6].

Now, (\mathbb{Q}, τ_p) , where τ_p is the particular point topology and $p \in \mathbb{Q}$ [11], is not S -normal. But, since the only closed domains are \emptyset and \mathbb{Q} , then it is mildly normal. Here is an example of an S -normal space which is not mildly normal.

Example 14. The modified Dieudonné plank X is S -normal, see Example 2. It is not mildly normal see [4]. ■

The following problems are still open:

- (1) Is S -normality hereditary with respect to closed subspaces?
- (2) If X is a Dowker space, is then $X \times I$ S -normal?
- (3) Does there exist an L -normal space which is not S -normal?

REFERENCES

- [1] E. K. van Douwen, *The Integers and Topology*, in: Kunen K. editor. Handbook of Set-Theoretic Topology, Amsterdam. the Netherlands: North-Holland. (1984) pp 111-167.
- [2] R. Engelking, *General Topology*, Warsaw. Poland: PWN. (1977)
- [3] L. Kalantan and M. Alhomieyed, *CC-normal Topological spaces*, Turk J of Math, 41, (2017) 749-755.
- [4] L. Kalantan and M. Saeed, *L-Normality*, Topology Proc., 50, (2017) 141-149.
- [5] L. Kalantan and P. Szeptycki, κ -*normality and products of ordinals*, Topol Appl. vol 123, 3, (2002) 537-545.
- [6] L. Kalantan, *Results about κ -normality*, Topol Appl. 125, (2002) 47-62.
- [7] L. Kalantan and S. AlZahrani, *C-normal Topological Property*, Filomat, 31:2, (2017) 407-411.
- [8] S. Mrówka, *On Completely Regular Spaces*, Fund. Math., 41, (1954) 105-106.
- [9] M. K. Singal and A. R. Singal, *Mildly Normal Spaces*, Kyungpook Math J. 13, (1973) 29-31.
- [10] E. V. Shchepin, *Real Valued Functions and Spaces Close to Normal*, Sib. J. Math. 13, (1972) 1182-1196.
- [11] L. Steen and J.A. Seebach, *Counterexamples in Topology*, Mineola. NY. USA: USA; Dover Publications. (1995)

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