SOME NEW DOUBLE SEQUENCE SPACES IN $n$-NORMED SPACES DEFINED BY A SEQUENCE OF ORLICZ FUNCTION

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Abstract. In the present paper we introduce some new double sequence spaces in $n$-normed spaces defined by a sequence of Orlicz function $\mathcal{M} = (M_{k,l})$ and also examined some properties of the resulting sequence spaces.

1. Introduction and Preliminary

The initial works on double sequences is found in Bromwich [2]. Later on it was studied by Hardy [9], Moricz [12], Moricz and Rhoades [13], Tripathy ([22], [23]), Basarir and Sonalcan [1] and many others. Hardy [9] introduced the notion of regular convergence for double sequences. The concept of paranormed sequences was studied by Nakano [15] and Simmons [21] at the initial stage. The concept of 2-normed spaces was initially developed by Gähler [5] in the mid of 1960's while that of $n$-normed spaces one can see in Misiak [14]. Since, then many others have studied this concept and obtained various results, see Gunawan ([6], [7]) and Gunawan and Mashadi [8]. By the convergence of a double sequences we mean the convergence in the Pringsheim sense i.e. a double sequence $x = (x_{k,l})$ has Pringsheim limit $L$ (denoted by $P-\lim x = L$) provided that given $\epsilon > 0$ there exists $n \in \mathbb{N}$ such that $|x_{k,l} - L| < \epsilon$ whenever $k, l > n$, see [16]. We shall write more briefly as $P$-convergent. The double sequence $x = (x_{k,l})$ is bounded if there exists a positive number $M$ such that $|x_{k,l}| < M$ for all $k$ and $l$. Let $l_\infty''$ the space of all bounded double sequences such that $\|x_{k,l}\|_{\infty,2} = \sup_{k,l} |x_{k,l}| < \infty$.

The idea of difference sequence spaces were introduced by Kizmaz [10] and he defined the sequence spaces

$$X(\Delta) = \left\{ x = (x_k) : (\Delta x_k) \in X \right\}$$

for $X = l_\infty$, $c$ or $c_0$, where $\Delta x = (\Delta x_k) = (x_k - x_{k+1})$ for all $k \in \mathbb{N}$.

Later, these difference sequence spaces were generalized by Et and Colak see [3]. In [3] Et and Colak generalized the above sequence spaces to the sequence spaces as follows:

$$X(\Delta^m) = \left\{ x = (x_k) : (\Delta^m x_k) \in X \right\}$$

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for $X = l_\infty$, $c$ or $c_0$, where $m \in \mathbb{N}$, $\Delta^0x = (x_k)$, $\Delta x = (x_k - x_{k+1})$,
\[
\Delta^mx = (\Delta^m x_k) = (\Delta^{m-1} x_k - \Delta^{m-1} x_{k+1}) \quad \text{for all} \ k \in \mathbb{N}.
\]
The generalized difference has the following binomial representation,
\[
\Delta^mx_k = \sum_{v=0}^{m} (-1)^v \binom{m}{v} x_{k+v}
\]
for all $k \in \mathbb{N}$.

An orlicz function $M : [0, \infty) \rightarrow [0, \infty)$ is a continuous, non-decreasing and convex function such that $M(0) = 0$, $M(x) > 0$ for $x > 0$ and $M(x) \rightarrow \infty$ as $x \rightarrow \infty$.

Lindenstrauss and Tzafriri [11] used the idea of Orlicz function to define the following sequence space,
\[
\ell_M = \left\{ x \in \mathbb{R} : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) < \infty \right\}
\]
which is called an Orlicz sequence space. Also $\ell_M$ is a Banach space with the norm
\[
\|x\| = \inf\left\{ \rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \leq 1 \right\}.
\]

Also, it was shown in [11] that every Orlicz sequence space $\ell_M$ contains a subspace isomorphic to $\ell_p(p \geq 1)$. The $\Delta_2$-condition is equivalent to $M(Lx) \leq LM(x)$, for all $L$ with $0 < L < 1$. An Orlicz function $M$ can always be represented in the following integral form
\[
M(x) = \int_0^x \eta(t)dt
\]
where $\eta$ is known as the kernel of $M$, is right differentiable for $t \geq 0$, $\eta(0) = 0$, $\eta(t) > 0$, $\eta$ is non-decreasing and $\eta(t) \rightarrow \infty$ as $t \rightarrow \infty$.

A double sequence space $E$ is said to be solid if $(a_{k,l}x_{k,l}) \in E$ whenever $(x_{k,l}) \in E$ and for all double sequences $(a_{k,l})$ of scalars with $|a_{k,l}| \leq 1$, for all $k,l \in \mathbb{N}$.

Let $n \in \mathbb{N}$ and $X$ be a linear space over the field $\mathbb{K}$, where $\mathbb{K}$ is field of real or complex numbers of dimension $d$, where $d \geq n \geq 2$. A real valued function $||\cdot, \ldots, \cdot||$ on $X^n$ satisfying the following four conditions:

1. $||x_1, x_2, \ldots, x_n|| = 0$ if and only if $x_1, x_2, \ldots, x_n$ are linearly dependent in $X$;
2. $||x_1, x_2, \ldots, x_n||$ is invariant under permutation;
3. $||\alpha x_1, x_2, \ldots, x_n|| = |\alpha| ||x_1, x_2, \ldots, x_n||$ for any $\alpha \in \mathbb{K}$, and
4. $||x + x', x_2, \ldots, x_n|| \leq ||x, x_2, \ldots, x_n|| + ||x', x_2, \ldots, x_n||$

is called a $n$-norm on $X$ and the pair $(X, ||\cdot, \ldots, \cdot||)$ is called a $n$-normed space over the field $\mathbb{K}$.

For example, we may take $X = \mathbb{R}^n$ being equipped with the $n$-norm $||x_1, x_2, \ldots, x_n||_E = \text{the volume of the } n\text{-dimensional parallelopiped spanned by the vectors } x_1, x_2, \ldots, x_n$ which may be given explicitly by the formula
\[
||x_1, x_2, \ldots, x_n||_E = |\det(x_{ij})|,
\]
where $x_i = (x_{i1}, x_{i2}, \ldots, x_{in}) \in \mathbb{R}^n$ for each $i = 1, 2, \ldots, n$. Let $(X, ||\cdot, \ldots, \cdot||)$ be an $n$-normed space of dimension $d \geq n \geq 2$ and $\{a_1, a_2, \ldots, a_n\}$ be linearly independent set in $X$. Then the following function $||\cdot, \ldots, \cdot||_\infty$ on $X^{n-1}$ defined by
\[
||x_1, x_2, \ldots, x_{n-1}||_\infty = \max\{||x_1, x_2, \ldots, x_{n-1}, a_i|| : i = 1, 2, \ldots, n\}$
defines an \((n - 1)\)-norm on \(X\) with respect to \(\{a_1, a_2, \cdots, a_n\}\).

A sequence \((x_k)\) in a \(n\)-normed space \((X, ||\cdot||, \cdots, ||\cdot||)\) is said to converge to some \(L \in X\) if

\[
\lim_{k \to \infty} ||x_k - L, z_1, \cdots, z_{n-1}|| = 0 \text{ for every } z_1, \cdots, z_{n-1} \in X.
\]

A sequence \((x_k)\) in a \(n\)-normed space \((X, ||\cdot||, \cdots, ||\cdot||)\) is said to be Cauchy if

\[
\lim_{k,p \to \infty} ||x_k - x_p, z_1, \cdots, z_{n-1}|| = 0 \text{ for every } z_1, \cdots, z_{n-1} \in X.
\]

If every Cauchy sequence in \(X\) converges to some \(L \in X\), then \(X\) is said to be complete with respect to the \(n\)-norm. Any complete \(n\)-normed space is said to be \(n\)-Banach space.

Let \(X\) be a linear metric space. A function \(p : X \to \mathbb{R}\) is called a paranorm, if

1. \(p(x) \geq 0\), for all \(x \in X\);
2. \(p(-x) = p(x)\), for all \(x \in X\);
3. \(p(x + y) \leq p(x) + p(y)\), for all \(x, y \in X\);
4. if \((\lambda_n)\) is a sequence of scalars with \(\lambda_n \to \lambda\) as \(n \to \infty\) and \((x_n)\) is a sequence of vectors with \(p(x_n - x) \to 0\) as \(n \to \infty\), then \(p(\lambda_n x_n - \lambda x) \to 0\) as \(n \to \infty\).

A paranorm \(p\) for which \(p(x) = 0\) implies \(x = 0\) is called total paranorm and the pair \((X, p)\) is called a total paranormed space. It is well known that the metric of any linear metric space is given by some total paranorm (see [24], Theorem 10.4.2, P-183). For more details about sequence spaces see ([4], [17], [18], [19]).

Let \((X, ||\cdot||, \cdots, ||\cdot||)\) be any \(n\)-normed space and let \(S''(n - X)\) denote \(X\)-valued sequence spaces. Clearly \(S''(n - X)\) is a linear space under addition and scalar multiplication.

Let \(M = (M_{k,l})\) be a sequence of Orlicz function and \((X, ||\cdot||, \cdots, ||\cdot||)\) an \(n\)-normed space. Let \(p = (p_{k,l})\) be a bounded sequence of positive real numbers and \(u = (u_{k,l})\) be any sequence of strictly positive real numbers. In the present paper, we define the following classes of double sequences:

\[
l''(M, \Delta^m, p, u, ||\cdot||, \cdots, ||\cdot||) = \left\{ x \in S''(n - X) : \sum_{k, l=1}^{\infty} u_{k,l} M_{k,l} \left( \left| \frac{\Delta^m x_{k,l}}{\rho}, z_1, z_2, \cdots, z_{n-1} \right| \right) \right\}^{p_{k,l}} < \infty,
\]

for each \(z_1, z_2, \cdots, z_{n-1} \in X\) and \(\rho > 0\).

If we take \(M(x) = x\), we get

\[
l''(\Delta^m, p, u, ||\cdot||, \cdots, ||\cdot||) = \left\{ x \in S''(n - X) : \sum_{k, l=1}^{\infty} \left[ u_{k,l} \left( \left| \frac{\Delta^m x_{k,l}}{\rho}, z_1, z_2, \cdots, z_{n-1} \right| \right) \right]^{p_{k,l}} < \infty,
\]

for each \(z_1, z_2, \cdots, z_{n-1} \in X\) and \(\rho > 0\).

If we take \(p = (p_{k,l}) = 1\), we have
\[ l''(\mathcal{M}, \Delta^m, u, ||\cdot, \cdot, \cdot||) = \]
\[ \left\{ x \in S''(n - X) : \sum_{k, l = 1}^{\infty, \infty} u_{k, l} M_{k, l} \left( \left\| \frac{\Delta^m x_{k, l}}{\rho}, z_1, z_2, \cdots, z_{n-1} \right\| \right) < \infty, \right. \]
\[ \text{for each } z_1, z_2, \cdots, z_{n-1} \in X \text{ and } \rho > 0 \} . \]

If we take \( m = 0, u = u_{k, l} = 1 \) and \( M_{k, l} = M \), we get the spaces which were defined and studied by E. Savas [20]. The work of this paper is motivated by the work of E. Savas [20], M. Basarir and O. Sonalcan [1], B. C. Tripathy [22], A. Esi [4] and H. Gunawan and M. Mashadi [8].

The following inequality will be used throughout the paper. Let \( p = (p_{k, l}) \) be a double sequence of positive real numbers with \( 0 < p_{k, l} \leq \sup_{k, l} = H \) and let \( K = \max\{1, 2^{H-1}\} \). Then for the factorable sequences \( \{a_{k, l}\} \) and \( \{b_{k, l}\} \) in the complex plane, we have
\[ |a_{k, l} + b_{k, l}|^{p_{k, l}} \leq K(|a_{k, l}|^{p_{k, l}} + |b_{k, l}|^{p_{k, l}}) . \] (1.1)

The aim of this paper is to introduce some new double sequence spaces in \( n \)-normed spaces defined by a sequence of Orlicz function \( \mathcal{M} = (M_{k, l}) \) and to establish some topological properties and some inclusion relation between above defined sequence spaces.

2. Main Results

**Theorem 2.1.** Let \( \mathcal{M} = (M_{k, l}) \) be a sequence of Orlicz function, \( p = (p_{k, l}) \) be a bounded sequence of positive real numbers and \( u = (u_{k, l}) \) be any sequence of strictly positive real numbers. Then \( l''(\mathcal{M}, \Delta^m, p, u, ||\cdot, \cdot, \cdot||) \) is a linear space over the field of complex number \( \mathbb{C} \).

**Proof.** Let \( x = (x_{k, l}), y = (y_{k, l}) \in l''(\mathcal{M}, \Delta^m, p, u, ||\cdot, \cdot, \cdot||) \) and \( \alpha, \beta \in \mathbb{C} \). Then there exist positive numbers \( \rho_1 \) and \( \rho_2 \) such that
\[ \sum_{k, l = 1}^{\infty, \infty} u_{k, l} M_{k, l} \left( \left\| \frac{\Delta^m x_{k, l}}{\rho_1}, z_1, z_2, \cdots, z_{n-1} \right\| \right)^{p_{k, l}} < \infty, \text{ for some } \rho_1 > 0, \]
\[ \sum_{k, l = 1}^{\infty, \infty} u_{k, l} M_{k, l} \left( \left\| \frac{\Delta^m y_{k, l}}{\rho_2}, z_1, z_2, \cdots, z_{n-1} \right\| \right)^{p_{k, l}} < \infty, \text{ for some } \rho_2 > 0 . \]

Define \( \rho_3 = \max(\alpha \rho_1, \beta \rho_2) \). Since \( ||\cdot, \cdot, \cdot|| \) is a \( n \)-norm on \( X \) and \( \mathcal{M} = (M_{k, l}) \) is a sequence of Orlicz function, we get
Let \( \mathcal{M} = (M_{k,l}, t) \) be a sequence of Orlicz function, \( p = (p_{k,l}) \) be a bounded sequence of positive real numbers and \( u = (u_{k,l}) \) be any sequence of strictly positive real numbers. Then \( l''(\mathcal{M}, \Delta^m, p, u, ||\cdot\cdot\cdot, ||) \) is a paranormed space with the paranorm defined by

\[
g(x) = \inf \left\{ \sum_{k, l=1}^{\infty} \left[ u_{k,l} M_{k,l} \left( \frac{\Delta^m x_{k,l}}{\rho}, z_1, z_2, \cdots, z_{n-1} \right) \right]^{p_{k,l}} : \left( \frac{\rho}{\rho'} \right)^{p_{k,l}} \leq 1 \right\},
\]

where \( 0 < p_{k,l} \leq \sup p_{k,l} = H, G = \max(1, H) \).

**Proof.** Clearly, \( g(\theta) = 0 \), where \( \theta = (0, 0, \cdots, 0) \) is the zero sequence and \( g(-x) = g(x) \). Let \( x = (x_{k,l}), y = (y_{k,l}) \in l''(\mathcal{M}, \Delta^m, p, u, ||\cdot\cdot\cdot, ||) \) and \( \alpha, \beta \in \mathbb{C} \). Then there exist positive numbers \( \rho_1 \) and \( \rho_2 \) such that

\[
\sum_{k, l=1}^{\infty} \left[ u_{k,l} M_{k,l} \left( \frac{\Delta^m x_{k,l}}{\rho_1}, z_1, z_2, \cdots, z_{n-1} \right) \right]^{p_{k,l}} < \infty, \text{ for some } \rho_1 > 0
\]

and

\[
\sum_{k, l=1}^{\infty} \left[ u_{k,l} M_{k,l} \left( \frac{\Delta^m y_{k,l}}{\rho_2}, z_1, z_2, \cdots, z_{n-1} \right) \right]^{p_{k,l}} < \infty, \text{ for some } \rho_2 > 0
\]

Let \( \rho = \rho_1 + \rho_2 \), we have
for sufficiently large value of \( k \) and thus \( x \) and thus \( g \). This implies that

\[
\text{Theorem 2.3.}
\]

\[
This gives us \( g(x + y) = \inf \left\{ \sum_{k, l} \left[ u_{k,l}M_{k,l}(\left| \frac{\Delta^n(x_k + y_k)}{\rho}, z_1, z_2, \ldots, z_{n-1} \right|) \right]^{p_{k,l}} \right\}^{\frac{1}{p}} \]

and thus

\[
g(\lambda x) = \inf \left\{ \sum_{k, l} \left[ u_{k,l}M_{k,l}(\left| \frac{\lambda \Delta^n x_k}{\rho}, z_1, z_2, \ldots, z_{n-1} \right|) \right]^{p_{k,l}} \right\}^{\frac{1}{p}} < \infty \}
\]

Now, let \( \lambda \to 0 \) and \( g(x^n - x) \to 0 \) as \( n \to \infty \). Since

\[
This gives us \( g(\lambda x^n) \to 0 \) as \( n \to \infty \).
\]

**Theorem 2.3.** If \( 0 < p_{k,l} < q_{k,l} < \infty \) for each \( k \) and \( l \), then

\[
l''(\mathcal{M}, \Delta^n, p, u, \left| \cdot \right|, \left| \cdot \right|) \subseteq l''(\mathcal{M}, \Delta^n, q, u, \left| \cdot \right|, \left| \cdot \right|).
\]

**Proof.** If \( x \in l''(\mathcal{M}, \Delta^n, p, u, \left| \cdot \right|, \left| \cdot \right|) \), then there exists some \( \rho > 0 \) such that

\[
\sum_{k, l} \left[ u_{k,l}M_{k,l}(\left| \frac{\Delta^n x_k}{\rho}, z_1, z_2, \ldots, z_{n-1} \right|) \right]^{p_{k,l}} < \infty.
\]

This implies that

\[
u_{k,l}M_{k,l}(\left| \frac{\Delta^n x_k}{\rho}, z_1, z_2, \ldots, z_{n-1} \right|) < 1,
\]

for sufficiently large value of \( k \) and \( l \). Since \( \mathcal{M} = (M_{k,l}) \) is non-decreasing, we have

\[
\sum_{k, l} \left[ u_{k,l}M_{k,l}(\left| \frac{\Delta^n x_k}{\rho}, z_1, z_1, \ldots, z_{n-1} \right|) \right]^{q_{k,l}} \]

\[
\leq \sum_{k, l} \left[ u_{k,l}M_{k,l}(\left| \frac{\Delta^n x_k}{\rho}, z_1, z_1, \ldots, z_{n-1} \right|) \right]^{p_{k,l}} < \infty.
\]

Thus \( x \in l''(\mathcal{M}, \Delta^n, q, u, \left| \cdot \right|, \left| \cdot \right|) \). This completes the proof.
Theorem 2.4. (i) If $0 < p_{k,l} < 1$ for each $k$ and $l$, then
\[ l''(\mathcal{M}, \Delta^m, p, u, ||\cdot||, \ldots, ||\cdot||) \subseteq l''(\mathcal{M}, \Delta^m, u, ||\cdot||, \ldots, ||\cdot||), \]

(ii) If $p_{k,l} \geq 1$ for each $k$ and $l$, then
\[ l''(\mathcal{M}, \Delta^m, u, ||\cdot||, \ldots, ||\cdot||) \subseteq l''(\mathcal{M}, \Delta^m, p, u, ||\cdot||, \ldots, ||\cdot||). \]

Proof. (i) Let $x = (x_{k,l}) \in l''(\mathcal{M}, \Delta^m, p, u, ||\cdot||, \ldots, ||\cdot||)$. Since $0 < \inf p_{k,l} < 1$, we have
\[
\infty \sum_{k,l=1}^{\infty} \left| u_{k,l} \right| M_{k,l} \left( \left| \frac{\Delta^m x_{k,l}}{\rho}, z_1, z_2, \ldots, z_{n-1} \right| \right) \leq \infty \sum_{k,l=1}^{\infty} \left| u_{k,l} \right| M_{k,l} \left( \left| \frac{\Delta^m x_{k,l}}{\rho}, z_1, z_2, \ldots, z_{n-1} \right| \right)^{p_{k,l}}
\]
and hence $x = (x_{k,l}) \in l''(\mathcal{M}, \Delta^m, p, u, ||\cdot||, \ldots, ||\cdot||)$.

(ii) Let $p_{k,l}$ for each $(k, l)$ and sup $p_{k,l} < \infty$. Let $x = (x_{k,l}) \in l''(\mathcal{M}, \Delta^m, u, ||\cdot||, \ldots, ||\cdot||)$. Then, for each $0 < \epsilon < 1$, there exists a positive integer $n$ such that
\[
\infty \sum_{k,l=1}^{\infty} \left| u_{k,l} \right| M_{k,l} \left( \left| \frac{\Delta^m x_{k,l}}{\rho}, z_1, z_2, \ldots, z_{n-1} \right| \right) \leq \epsilon < 1,
\]
for all $k, l \in \mathbb{N}$. This implies that
\[
\infty \sum_{k,l=1}^{\infty} \left| u_{k,l} \right| M_{k,l} \left( \left| \frac{\Delta^m x_{k,l}}{\rho}, z_1, z_2, \ldots, z_{n-1} \right| \right)^{p_{k,l}} \leq \infty \sum_{k,l=1}^{\infty} \left| u_{k,l} \right| M_{k,l} \left( \left| \frac{\Delta^m x_{k,l}}{\rho}, z_1, z_2, \ldots, z_{n-1} \right| \right).
\]
Thus $x = (x_{k,l}) \in l''(\mathcal{M}, \Delta^m, p, u, ||\cdot||, \ldots, ||\cdot||)$ and this completes the proof.

Theorem 2.5. Let $\mathcal{M}' = (M'_{k,l})$ and $\mathcal{M}'' = (M''_{k,l})$ be sequences of Orlicz function. Then
\[ l''(\mathcal{M}', \Delta^m, p, u, ||\cdot||, \ldots, ||\cdot||) \cap l''(\mathcal{M}'', \Delta^m, p, u, ||\cdot||, \ldots, ||\cdot||) \subseteq l''(\mathcal{M}' + \mathcal{M}'', \Delta^m, p, u, ||\cdot||, \ldots, ||\cdot||). \]

Proof. We have
\[
\left[ u_{k,l}(M'_{k,l} + M''_{k,l}) \left( \left| \frac{\Delta^m x_{k,l}}{\rho}, z_1, z_2, \ldots, z_{n-1} \right| \right) \right]^{p_{k,l}}
\leq K \left[ u_{k,l} M'_{k,l} \left( \left| \frac{\Delta^m x_{k,l}}{\rho}, z_1, z_2, \ldots, z_{n-1} \right| \right) \right]^{p_{k,l}}
+ K \left[ u_{k,l} M''_{k,l} \left( \left| \frac{\Delta^m x_{k,l}}{\rho}, z_1, z_2, \ldots, z_{n-1} \right| \right) \right]^{p_{k,l}}.
\]
Let $x = (x_{k,l}) \in l''(\mathcal{M}', \Delta^m, p, u, ||\cdot||, \ldots, ||\cdot||) \cap l''(\mathcal{M}'', \Delta^m, p, u, ||\cdot||, \ldots, ||\cdot||)$, when adding the above inequality from $k, l = 0, 0$ to $\infty, \infty$ we get $x = (x_{k,l}) \in l''(\mathcal{M}' + \mathcal{M}'', \Delta^m, p, u, ||\cdot||, \ldots, ||\cdot||)$ and this completes the proof.
Theorem 2.6. The sequence space \( l''(\mathcal{M}, \Delta^m, p, u, ||\cdot||) \) is solid.

Proof. Let \( x = (x_{k,l}) \in l''(\mathcal{M}, \Delta^m, p, u, ||\cdot||) \), i.e.
\[
\sum_{k, l=1}^{\infty} \left[ u_{k,l} M_{k,l} \left( \left\| \Delta^m x_{k,l} / \rho, z_1, z_2, \ldots, z_{n-1} \right\| \right) \right]^{p_{k,l}} < \infty.
\]

Let \( (\alpha_{k,l}) \) be double sequence of scalars such that \( |\alpha_{k,l}| \leq 1 \) for all \( k, l \in \mathbb{N} \times \mathbb{N} \). Then, the result follows from the following inequality
\[
\sum_{k, l=1}^{\infty} \left[ u_{k,l} M_{k,l} \left( \left\| \Delta^m x_{k,l} / \rho, z_1, z_2, \ldots, z_{n-1} \right\| \right) \right]^{p_{k,l}} 
\leq \sum_{k, l=1}^{\infty} \left[ u_{k,l} M_{k,l} \left( \left\| \Delta^m x_{k,l} / \rho, z_1, z_2, \ldots, z_{n-1} \right\| \right) \right]^{p_{k,l}},
\]
and this completes the proof. \( \square \)

Theorem 2.7. The sequence space \( l''(\mathcal{M}, \Delta^m, p, u, ||\cdot||) \) is monotone.

Proof. It is obvious. \( \square \)

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