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EXACT SOLUTIONS OF HYDROMAGENTIC FLUID FLOW ALONG AN INCLINED PLANE WITH HEAT AND MASS TRANSFER

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Abstract. The major focus of this research is to discuss the precise results of the unstable, indirect and natural free convection flow of radiating fluid with heat and the analysis of its collective transmission along with an inclined plane. The subsequent liquid is optically thick, sticky and is unable to be compressed and further it is prone to electrical conducting. Analytical solutions are developed by applying the method of Laplace transform for the governing differential equations. The obtained results are discussed minutely for some special cases and further the effects of the related parameters are considered in detail. Rosseland radiational flux model is employed in order to model the impacts of the thermal radiation. For this purpose, graphical and numerical results are derived for the appropriate parameters in flow regime and these parameters exert significant impact on the obtained results.

1. Introduction

The hydromagnetic convection flow past a vertical plate has been considered in many problems in the previous studies due to its practical applications in many fields of engineering and scientific problems. Numerous investigations were implemented using numerical and analytical approaches with thermal conditions of different type which are well defined and continuous at the wall. This is because of the fact that impact of radiation on convection is reasonably imperative in the framework of various applications for example in transportation system, Pollution dispersion, cooling and heating of channels, Biological sciences, gas turbines and nuclear reactors, electrical power generation and also many other fields. Among such applications the heat and mass transfer combinations for free convective processes are analyzed under various processes, like in chemical processing systems along with various industrial applications. Thus, Magneto-Hydrodynamics show much importance in cosmological and environmental fields, nuclear and chemical engineering and in electronics for freeconvective flows. Such wide applicability of magnetic field in fluid dynamic has been analyzed by various scientists under different conclusions. The analytical solution and physical aspects of heat and mass

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transfer past an inclined oscillating surface under the impact of mass diffusion and thermal radiation investigated by [1]. The unsteady free convection flow with heat and mass transfer of an electrically conducting viscoelastic fluid through a porous medium of variable permeability was investigated [2]. They use Laplace Transform and method of separation of variables to explore the impact of various important parameters. Moreover, it was found that, if the applied magnetic field is fixed to the plate the fluid flows more slowly than if the magnetic field is fixed to fluid. If the magnetic field strength is high, the fluid moves more slowly than into weak magnetic fields. The fully-developed transient free-convection flow of viscous reactive fluid problem in a vertical tube is analyzed both analytically and numerically [3]. This study reports the effect of several operating parameters on the flow hydrodynamics and thermal characteristics. They obtained solutions for transient state velocity and temperature fields by implicit finite difference method and perturbation series method. Norfifah Bachok et al [4] investigated the heat transfer for moving plates, Sadia and Hossain [5] deliberated the mixed convention boundary layer flow with mass and heat transfer having variable viscosity. The transport processes, which are important for industrial applications, in which mass and heat transfers were considered by Ellahi [6] in recent past, who discussed the MagnetoHydrodynamics effects over nano fluid in a pipe having variable viscosity. The thermal radiational effect on hydromagnetic Couette flow was presented by [7] and [8]. They used a flux model to investigate radiational magneto- hydrodynamic channel flow. The steady radiative magneto gas dynamics Couette flow for variable coefficients of viscosity and density-dependent absorption coefficient has been described by [9]. He also numerically computed velocity, radiative flux, induced magnetic field and temperature profiles. This shows that wall electrical conductivity and emissivity exert a major effect on the velocity and magnetic field distributions but have minor influence on temperatures. The fully developed mixed convection flow in a vertical channel filled with nanofluids in the presence of a uniform transverse magnetic field has been studied [10]. Closed form solutions for the fluid temperature, velocity and induced magnetic field are obtained for both the buoyancy-aided and -opposed flows. The unsteady magneto-hydrodynamic radiational convective flow with the use of Rosseland model for radiation was reviewed by [11]. Analytical solutions for temperature induced magnetic field and velocity has also been discussed by them. The radiational effect of magnetohydrodynamics flow of gas between two concentric spheres employing the optically thin limit case for thermal radiation is studied by [12]. On the other hand, numerous practical interest problems of which might contain non-uniform conditions at the plates. Considering this fact, some researchers, namely, Makinde [13] , Pal et al [14], Sarma, D. and K. Pandit [15], Usman et al [16] studied natural convection flow from a vertical plate along with surface temperature by seeing different characteristics of the problems. Patra et al. and Seth [17, 18] explored the impacts of radiation on natural convection flow of an incompressible and viscous fluid near avertical plate. Azzam [19], used the Rosseland approximation to explain the impact of radiation flux on hydromagnetic free forced steady and laminar boundary layer flow. Analytically solutions was obtained for free convectional magneto-heat transfer [20], using the differential approximation for energy equation with radiation flux and viscous effects. A steady, laminar, magneto-hydrodynamic and convectional flow with thermal mass and heat transmission and a semi-infinite plate were considered by [21], the model solutions are obtained numerically. Mahmoud [22] studied the temperature-dependent viscosity effects in transient dissipative radiation-hydromagnetic convection and [23] discussed the numerical results of magneto hydrodynamic boundary layer flow for porous medium. The series solutions was obtained for unsteady squeezing flow between parallel plates with transmission of heat and mass [24]. The unsteady natural convectional flow of optically thick incompressible viscous fluid for vertical plates with temperature distribution and thermal diffusion effects were discussed by [25, 26]. Under the light of above researches, the main point of this research is to explore the unsteady, thermal, natural convectional fluid flow for an electrically conducting fluid with mass and heat transfer along an inclined plane. Moreover, frictional shearing stress, concentration gradient and temperature gradients at the plate surface are calculated for Grashof number, modified Grashof number, Schmidt number and Boltzmann Rosseland constant. For this purpose, analytical solutions are developed, and effects of the pertinent parameters are considered in detail. In order to prove the concerned point, this research has been arranged as follows: Section 2, describes mathematical formulation of problem along with the description of boundary conditions. In Section 3, Laplace Transform Method is used to derive the analytical results of proposed problem for different cases. Some special cases of the peresnnt study are discussed in Section 4. Further, the graphical and numerical results are explained in detail in Section 5. Finally, conclusion and remarks of this study are given in Section 6.

2. Mathematical Processing of the Problem

Suppose an incompressible viscous, transient hydromagnetic flow is under research for an electrically conducting, non-scattering, absorbing-emitting fluid with mass and heat transmission along an infinite plate making an angle α to the horizontal side. The velocity of the plate is and x, $y-$axis are considered axes along and perpendicular to the plate as shown in Figure 1.

A uniform magnetic field is applied perpendicular to the direction of plate is known as B_0. By using Maxwell field equations described by Sutton and Sherman [27], which are the result of the compromise of five vector equations, known as Amperes law, Ohms law, magnetic field continuity, Kirchhoffs law and Faraday s law. They described the propagation and interaction of electric and magnetic fields, influenced by objects. Thus, the generalized vector forms these Maxwell equations for an electrically conducting gas flow are

$$\nabla . B = 0, \tag{2.1}$$

$$\nabla . J = 0, \tag{2.2}$$

$$\nabla \times B = \mu J, \tag{2.3}$$

$$\nabla \times E = -\frac{\partial B}{\partial t}, \tag{2.4}$$

$$J = \sigma(\nu \times B + E), \tag{2.5}$$

where μ is known as viscosity and t is time, J is known as the current density, σ electrical conductivity of the fluid, B magnetic field vector, E electrical field intensity vector, v velocity vector, ρ is the density. For two-dimensional magneto hydro

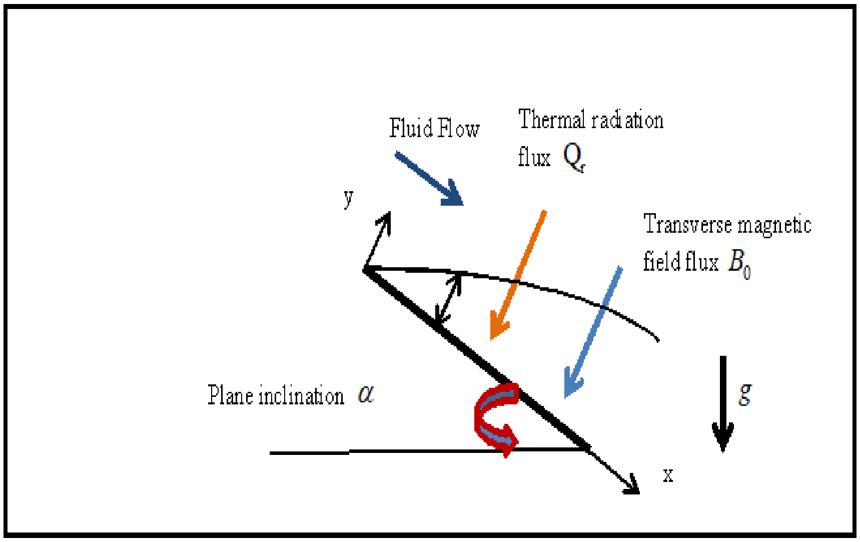

FIGURE 1. Flow geometry

dynamic and dynamic flow of fluid, the hydro magnetic retarding force applied only in the anti-parallel direction to the flow of fluid and can be written as

$$F_{magnetic} \approx -\sigma B_y^2 u, \tag{2.6}$$

where B_y is the component of magnetic field vector in the direction of y-axis. In mass and heat transfer processes, the concentration and temperature differences are small enough. The Boussines q approximation and the heat and mass transfer processes are analogously applicable. Under these simplifications, the unsteady indirect natural free convection flow for radiating fluid, by using Boussines q approximation, the mathematical equations are as under

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.7}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - \nu\frac{\partial^2 u}{\partial y^2} + \frac{\sigma B_0^2 u}{\rho} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + g\beta(T_f - T_\infty)\sin\alpha + g\beta_c(C - C_\infty), \tag{2.8}$$

$$-\frac{1}{\rho}\frac{\partial P}{\partial x} = g\beta(T_f - T_\infty)\cos\alpha, \tag{2.9}$$

$$\frac{\partial C}{\partial t} = D_1\frac{\partial^2 C}{\partial y^2}, \tag{2.10}$$

$$\frac{\partial T_f}{\partial t} = \frac{k}{\rho C_p}\frac{\partial^2 T_f}{\partial y^2} - \frac{1}{\rho C_p}\frac{\partial Q_r}{\partial y}, \tag{2.11}$$

Thus, the corresponding conditions are

$$u = u_0, T_f = T_w, t \geq 0, \text{ when } y \geq 0, \tag{2.12}$$

$$u = 0, T_f = T_\infty, t \leq 0, \text{ when } y = 0, \tag{2.13}$$

$$u \to 0, T_f = T_\infty, t \geq 0, \text{ when } y \to \infty, \tag{2.14}$$

where u and v are the components of velocity along with x-axis and y-axis, t is for the time, T_f is temperature of fluid, T_∞ is temperature of free stream, T_w is the plate surface temperature, μ and ν are fluid dynamic and kinematic viscosity, g is the gravitational acceleration, Q_r is radiative heat flux, β is thermal expansion coefficient, k and σ are thermal and electrical conductivities of the fluid, ρ is density, B_0 is magnetic field, C is fluid concentration and C_p is the specific heat, C_∞ is uniform ambient concentration, and D_1 is diffusion coefficient. And

$$P = \rho\beta g(h - y)(T_f - T_\infty)\cos(\alpha), \tag{2.15}$$

where h is known as an elevation, differentiating the above Eq. (15) yields

$$\frac{\partial P}{\partial x} = \rho g\beta(T_f - T_\infty)\frac{\partial h}{\partial x}\cos\alpha. \tag{2.16}$$

The special position at plate (x=0), the density gradient with depth is constant and hence the prescribed condition is as follows

$$\frac{\partial h}{\partial x} = \text{constant} = F_1, \tag{2.17}$$

To facilitate the solution of two-point boundary value problem (BVP) which is defined above, under the boundary conditions (12), (13), and (14) the dimensionless variables are

$$\begin{aligned} u^* &= \frac{u}{u_0}, t^* = t\frac{u_0^2}{\nu}, y^* = \frac{yu_0}{\nu}, \theta = \frac{(T_f - T_\infty)}{(T_w) - T_\infty}, \mathrm{Gr} = \frac{g\beta\nu(T_w - T_\infty)}{u_0^3}, \mathrm{Pr} = \frac{\mu C_P}{k}, \\ M &= \frac{\sigma\nu B_0^2}{\rho u_0^2}, \phi = \frac{(C - C_\infty)}{C_w - C_\infty}, \mathrm{Sc} = \frac{\nu}{\rho}, \mathrm{Gm} = \frac{g\beta_c\nu(C_w - C_\infty)}{u_0^3}, \end{aligned} \tag{2.18}$$

where θ is the dimensionless temperature function, Grashof number (free convection parameter) Gr, Gm is the modified Grashof number, Schmidt number Sc, Hartman number M, and Prandtl number Pr. Using transformation Eq. (18) into dimensionless variables, dropping $*$, and neglecting convective acceleration term, the result comes as follows

$$\frac{\partial u}{\partial t} = G_r(\theta\sin\alpha - F_1\theta\cos\alpha) + \frac{\partial^2 u}{\partial y^2} - M^2 u + G_m(\sin\alpha - F_1\cos\alpha)\phi. \tag{2.19}$$

Presently, the optically thick radiational limit is taken in order to simplify the problem. By using Rosseland radiation approximation formula by Raptis et al. [28], the Q_r is given as

$$Q_r = -\frac{1.333\sigma}{k^*}\frac{\partial T_f^4}{\partial y}, \tag{2.20}$$

where σ represents the Stefan-Boltzman constant and k^* is absorption parameter for the medium

$$T_f^4 = 4T_\infty^3 T_f - 3T_\infty^4. \tag{2.21}$$

Proceeding with the above analysis, the energy equation in dimensionless form is

$$[1 + k_1]\frac{\partial^2\theta}{\partial y^2} - \mathrm{Pr}\frac{\partial\theta}{\partial t} = 0, \tag{2.22}$$

$$\frac{\partial^2\phi}{\partial y^2} - \mathrm{Sc}\frac{\partial\phi}{\partial t} = 0, \tag{2.23}$$

where

$$k_1 = \frac{16\sigma T_\infty^3}{3kk^*}, \tag{2.24}$$

k_1 denotes the Boltzmann-Rosseland's radiatonal-conductive number and has different values corresponding to different phenomena. As ($k_1 > 1$), shows thermal radiation domination and for ($k_1 < 0$) corresponds to thermal condition domination. In the case of $k_1 = 1$ then both radiation and deduction dominance heat transfer modes will equally contribute to the system. The above conditions have also been transformed as under

$$u = 0, \theta = 0, \phi = 0, t \leq 0 \text{ when } y \geq 0, \tag{2.25}$$

$$u = 1, \theta = 1, \phi = 1, t > 0 \text{ when } y = 0, \tag{2.26}$$

$$u \to 0, \theta \to 0, \phi \to 0, t > 0 \text{ when } y \to \infty. \tag{2.27}$$

3. Analytical Solution

In this section an attempt is made to derive the analytical solutions of thee proposed mathematical model by using Laplace Transform Method. This method solves differential equations along with initial and boundary conditions. Applying Laplace transform technique on these equations (19), (22) and (23) along with conditions (25), (26), and (27), gives the results as

$$\begin{aligned} s\bar{u}(y,s) - u(y,0) \quad &= \quad \mathrm{Gr}(\sin\alpha\theta(y,s) - F_1\cos\alpha\theta(y,s)) + \frac{\partial^2\bar{u}(y,s)}{\partial y^2} - \\ &\quad\; M^2\bar{u}(y,s) + \mathrm{Gm}(\sin\alpha - F_1\cos\alpha)\bar{\phi}(y,s) \end{aligned} \tag{3.1}$$

$$[1 + k_1]\frac{\partial^2\bar{\theta}(y,s)}{\partial y^2} = \mathrm{Pr}[s\bar{\theta}(y,s) - \theta(y,0)] \tag{3.2}$$

$$\frac{\partial^2\bar{\phi}(y,s)}{\partial y^2} = \mathrm{Sc}[s\bar{\phi}(y,s) - \phi(y,0)] \tag{3.3}$$

The Laplace transforms of *u(y,t)*, $\theta(y,t)$ and $\phi(y,t)$ are represented by $\bar{u}(y,s)$, $\bar{\theta}(y,s)$ and $\bar{\phi}(y,s)$ respectively, and defined as

$$\begin{aligned} \bar{u}(y,s) &= \int_0^\infty \exp(-st)u(y,t)dt, \\ \bar{\theta}(y,s) &= \int_0^\infty \exp(-st)\theta(y,t)dt, \\ \bar{\phi}(y,s) &= \int_0^\infty \exp(-st)\phi(y,t)dt, \end{aligned} \tag{3.4}$$

$$\begin{aligned} &\frac{\partial^2\bar{u}(y,s)}{\partial y^2} - s\bar{u}(y,s) + \mathrm{Gr}(\sin\alpha\theta(y,s) - F_1\cos\alpha\theta(y,s)) - \\ &M^2\bar{u}(y,s) + \mathrm{Gm}(\sin\alpha - F_1\cos\alpha)\bar{\phi}(y,s) = 0, \end{aligned} \tag{3.5}$$

$$\frac{\partial^2 \bar{\theta}(y,s)}{\partial y^2} - s\bar{\theta}(y,s)\frac{\mathrm{Pr}}{1+k_1} = 0, \tag{3.6}$$

$$\frac{\partial^2 \phi(y,s)}{\partial y^2} - \mathrm{Sc}s\bar{\phi}(y,s) = 0. \tag{3.7}$$

The corresponding transformed boundary conditions are

$$\begin{aligned} &\bar{u}(y,s) = \frac{1}{s}, \bar{\theta}(y,s) = \frac{1}{s}, \bar{\phi}(y,s) = \frac{1}{s}, \\ &\bar{u}(y,s) \to 0, \bar{\theta}(y,s) \to 0, \bar{\phi}(y,s) \to 0, t > 0 \text{ when } y \to \infty. \end{aligned} \tag{3.8}$$

Applying inverse Laplace transform, we get the solution of Eqs. (32) to (34) with respect to the boundary condition (35). The simplified solution becomes

$$
\begin{aligned}
&u(y,t)=\frac{1}{2}\left[\exp(My)\mathrm{erfc}\left(\frac{0.5y}{\sqrt{t}}+M\sqrt{t}\right)+\exp(-My)\mathrm{erfc}\left(\frac{y}{2\sqrt{t}}-M\sqrt{t}\right)\right]\\
&\left[1-\frac{(\mathrm{Gr}+\mathrm{Gm})(\sin\alpha-F_1\cos\alpha)}{M^2}\right]\\
&+\frac{0.5\mathrm{Gr}(\sin\alpha-F_1\cos\alpha)}{M^2}\exp\left(\frac{M^2(1+k_1)t}{\mathrm{Pr}-1-k_1}\right)\\
&\left[\exp\left(My\sqrt{\frac{\mathrm{Pr}}{\mathrm{Pr}-1-k_1}}\right)\mathrm{erfc}\left(\frac{0.5y}{\sqrt{t}}+M\sqrt{\frac{\mathrm{pr}}{\mathrm{pr}-1-k_1}t}\right)\right.\\
&\left.+\exp\left(-My\sqrt{\frac{\mathrm{Pr}}{\mathrm{Pr}-1-k_1}}\right)\mathrm{erfc}\left(\frac{0.5y}{\sqrt{t}}-M\sqrt{\frac{\mathrm{pr}}{\mathrm{pr}-1-k_1}}\right)\right]+\\
&\frac{0.5\mathrm{Gm}(\sin\alpha-F_1\cos\alpha)}{M^2}\exp\left(\frac{M^2}{\mathrm{Sc}-1}t\right)\\
&\left[\exp\left(My\sqrt{\frac{\mathrm{Sc}}{\mathrm{Sc}-1}}\right)\mathrm{erfc}\left(\frac{.5y}{\sqrt{t}}+M\sqrt{\frac{\mathrm{Sc}}{\mathrm{Sc}-1}t}\right)\right.\\
&\left.+\exp\left(-My\sqrt{\frac{\mathrm{Sc}}{\mathrm{Sc}-1}}\right)\mathrm{erfc}\left(\frac{.5y}{\sqrt{t}}-M\sqrt{\frac{\mathrm{Sc}t}{\mathrm{Sc}-1}t}\right)\right]\\
&+\frac{(\sin\alpha-F_1\cos\alpha)}{M^2}\left[\mathrm{Gmerfc}\left(0.5y\sqrt{\frac{\mathrm{Sc}}{t}}\right)+0.5\mathrm{Gr}y\left(\sqrt{\frac{\mathrm{Pr}}{(1+k_1)t}}\right)\right]\\
&+\frac{(\sin\alpha-F_1\cos\alpha)}{M^2}\left[\mathrm{Gmerfc}\left(0.5y\sqrt{\frac{\mathrm{Sc}}{t}}\right)+0.5\mathrm{Gr}y\left(\sqrt{\frac{\mathrm{Pr}}{(1+k_1)t}}\right)\right]\\
&-\frac{.5\mathrm{Gr}(\sin\alpha-F_1\cos\alpha)}{M^2}\exp\left(\frac{M^2(1+k_1)t}{\mathrm{Pr}-1-k_1}\right)\exp\left(My\sqrt{\frac{\mathrm{Pr}}{\mathrm{Pr}-1-k_1}}\right)\\
&\mathrm{erfc}\left[\left(0.5y\sqrt{\frac{\mathrm{Pr}}{1+k_1t}}+M\sqrt{\frac{(1+k_1)t}{\mathrm{Pr}-1-k_1}}\right)+\exp\left(-My\sqrt{\frac{\mathrm{Pr}}{\mathrm{Pr}-1-k_1}}\right)\right.\\
&\left.\mathrm{erfc}\left(0.5y\sqrt{\frac{\mathrm{Pr}}{(1+k_1)t}}-M\sqrt{\frac{(1+k_1)t}{\mathrm{Pr}-1-k_1}}\right)\right]\frac{\mathrm{Gm}(\sin\alpha-F1\cos\alpha)}{M^2}\\
&\left[0.5\exp(\frac{M^2t}{\mathrm{Sc}-1})\left\{\exp\left(yM\sqrt{\frac{\mathrm{Sc}}{\mathrm{Sc}-1}}\right)\mathrm{erfc}\left(0.5y\sqrt{\frac{\mathrm{Sc}}{t}}+M\sqrt{\frac{t}{\mathrm{Sc}-1}}\right)\right.\right.\\
&\left.\left.+\exp\left(-yM\sqrt{\frac{\mathrm{Sc}}{\mathrm{Sc}-1}}\right)\mathrm{erfc}\left(0.5y\sqrt{\frac{\mathrm{Sc}}{t}}-M\sqrt{\frac{t}{\mathrm{Sc}-1}}\right)\right\}\right],
\end{aligned}
\tag{3.9}
$$

and

$$\theta(y,t)=\mathrm{erfc}\left(0.5y\sqrt{\frac{\mathrm{Pr}}{(1+k_1)t}}\right), \tag{3.10}$$

$$\phi(y,t)=\mathrm{erfc}\left(0.5y\sqrt{\frac{\mathrm{Sc}}{t}}\right). \tag{3.11}$$

4. Special Cases

Several special cases can be extracted from the above derived solution

4.1. **Case 1. Reduction of thermal effects.** When the thermal radiation affect $K_1 \to 0$, then

$$
\begin{aligned}
&u(y,t) = \frac{1}{2}\left[\mathrm{e}^{My}\mathrm{erfc}\left(\frac{0.5y}{\sqrt{t}} + M\sqrt{t}\right) + \mathrm{e}^{-My}\mathrm{erfc}\left(\frac{y}{2\sqrt{t}} - M\sqrt{t}\right)\right]\\
&\left[1 - \frac{(\mathrm{Gr}+\mathrm{Gm})(\sin\alpha - F_1\cos\alpha)}{M^2}\right] + \frac{0.5\mathrm{Gr}(\sin\alpha - F_1\cos\alpha)}{M^2}\exp\left(\frac{M^2 t}{\mathrm{Pr}-1}\right)\\
&\left[\exp\left(My\sqrt{\frac{\mathrm{Pr}}{\mathrm{Pr}-1}}\right)\mathrm{erfc}\left(\frac{0.5y}{\sqrt{t}} + M\sqrt{\frac{\mathrm{Pr}}{\mathrm{Pr}-1}t}\right) + \right.\\
&\left.\exp\left(-My\sqrt{\frac{\mathrm{Pr}}{\mathrm{Pr}-1}}\right)\mathrm{erfc}\left(\frac{0.5y}{\sqrt{t}} - M\sqrt{\frac{\mathrm{Pr}}{\mathrm{Pr}-1}}\right)\right]\\
&+\frac{0.5\mathrm{Gm}(\sin\alpha - F_1\cos\alpha)}{M^2}\exp\left(\frac{M^2}{\mathrm{Sc}-1}t\right)\left[\exp\left(My\sqrt{\frac{\mathrm{Sc}}{\mathrm{Sc}-1}}\right)\mathrm{erfc}\left(\frac{0.5y}{\sqrt{t}} + M\sqrt{\frac{\mathrm{Sc}}{\mathrm{Sc}-1}t}\right)\right.\\
&\left.+\exp\left(-My\sqrt{\frac{\mathrm{Sc}}{\mathrm{Sc}-1}}\right)\mathrm{erfc}\left(\frac{0.5y}{\sqrt{t}} - M\sqrt{\frac{\mathrm{Sc}t}{\mathrm{Sc}-1}t}\right)\right]\\
&+\frac{(\sin\alpha - F_1\cos\alpha)}{M^2}\left[\mathrm{Gmerfc}\left(0.5y\sqrt{\frac{\mathrm{Sc}}{t}}\right) + 0.5\mathrm{Gr}y\sqrt{\frac{\mathrm{Pr}}{t}}\right] + \frac{(\sin\alpha - F_1\cos\alpha)}{M^2}\\
&\left[\mathrm{Gmerfc}\left(0.5y\sqrt{\frac{\mathrm{Sc}}{t}}\right) + 0.5\mathrm{Gr}y\sqrt{\frac{\mathrm{Pr}}{t}}\right] - \frac{0.5\mathrm{Gr}(\sin\alpha - F_1\cos\alpha)}{M^2}\exp\left(\frac{M^2 t}{\mathrm{Pr}-1}\right)\\
&\left[\exp\left(My\sqrt{\frac{\mathrm{Pr}}{\mathrm{Pr}-1}}\right) + \mathrm{erfc}\left(0.5y\sqrt{\frac{\mathrm{Pr}}{t}} + M\sqrt{\frac{(t)}{\mathrm{Pr}-1}}\right)\right]\\
&+\exp\left(-My\sqrt{\frac{\mathrm{Pr}}{\mathrm{Pr}-1}}\right)\mathrm{erfc}\left(0.5y\sqrt{\frac{\mathrm{Pr}}{(1+k_1)t}} - M\sqrt{\frac{(1+k_1)t}{\mathrm{Pr}-1}}\right) + \frac{\mathrm{Gm}(\sin\alpha - F1\cos\alpha)}{M^2}\\
&\left[0.5\exp\left(\frac{M^2 t}{\mathrm{Sc}-1}\right)\left\{\exp\left(yM\sqrt{\frac{\mathrm{Sc}}{\mathrm{Sc}-1}}\right)\mathrm{erfc}\left(0.5y\sqrt{\frac{Sc}{t}} + M\sqrt{\frac{t}{\mathrm{Sc}-1}}\right)\right.\right.\\
&\left.\left.+\exp\left(-yM\sqrt{\frac{\mathrm{Sc}}{\mathrm{Sc}-1}}\right)\mathrm{erfc}\left(0.5y\sqrt{\frac{\mathrm{Sc}}{t} - M\sqrt{\frac{t}{\mathrm{Sc}-1}}}\right)\right\}\right]. \qquad (4.1)
\end{aligned}
$$

4.2. **For Horizontal Convection.** When $\alpha \to 0, \sin\alpha \to 0, \cos\alpha \to 1$

$$
\begin{aligned}
&u(y,t) = \frac{1}{2}\left[\mathrm{e}^{My}\mathrm{erfc}\left(\frac{0.5y}{\sqrt{t}} + M\sqrt{t}\right) + \mathrm{e}^{-My}\mathrm{erfc}\left(\frac{y}{2\sqrt{t}} - M\sqrt{t}\right)\right] \\
&\left[1 + \frac{(\mathrm{Gr}+\mathrm{Gm})F_1}{M^2}\right] - \frac{0.5\mathrm{Gr}F_1}{M^2}\exp\left(\frac{M^2(1+k_1)t}{\mathrm{Pr}-1-k_1}\right) \\
&[\exp\left(My\sqrt{\frac{\mathrm{Pr}}{\mathrm{Pr}-1-k_1}}\right)\mathrm{erfc}\left(\frac{0.5y}{\sqrt{t}} + M\sqrt{\frac{\mathrm{Pr}}{\mathrm{Pr}-1-k_1}t}\right) \\
&+\exp\left(-My\sqrt{\frac{\mathrm{Pr}}{\mathrm{Pr}-1-k_1}}\right)\mathrm{erfc}\left(\frac{0.5y}{\sqrt{t}} - M\sqrt{\frac{\mathrm{Pr}}{\mathrm{Pr}-1-k_1}}\right) \\
&-\frac{0.5G_mF_1}{M^2}\exp\left(\frac{M^2}{\mathrm{Sc}-1}t\right)\left[\exp\left(My\sqrt{\frac{\mathrm{Sc}}{\mathrm{Sc}-1}}\right)\mathrm{erfc}\left(\frac{0.5y}{\sqrt{t}} + M\sqrt{\frac{\mathrm{Sc}}{\mathrm{Sc}-1}t}\right)\right. \\
&\left.+\exp\left(-My\sqrt{\frac{\mathrm{Sc}}{\mathrm{Sc}-1}}\right)\mathrm{erfc}\left(\frac{.5y}{\sqrt{t}} - M\sqrt{\frac{\mathrm{Sc}t}{\mathrm{Sc}-1}t}\right)\right] - \frac{-F_1}{M^2}\left[\mathrm{Gmerfc}\left(0.5y\sqrt{\frac{\mathrm{Sc}}{t}}\right)\right. \\
&\left.+0.5\mathrm{Gr}y\sqrt{\frac{\mathrm{Pr}}{(1+k_1)t}}\right] - \frac{F_1}{M^2}\left[\mathrm{Gmerfc}\left(0.5y\sqrt{\frac{\mathrm{Sc}}{t}}\right) + 0.5\mathrm{Gr}y\sqrt{\frac{\mathrm{Pr}}{(1+k_1)t}}\right] \\
&+\frac{.5\mathrm{Gr}F_1}{M^2}\exp\left(\frac{M^2(1+k_1)t}{\mathrm{Pr}-1-k_1}\right)\left[\exp\left(My\sqrt{\frac{\mathrm{Pr}}{\mathrm{Pr}-1-k_1}}\right)\right. \\
&\mathrm{erfc}\left(0.5y\sqrt{\frac{\mathrm{Pr}}{1+k_1t}} + M\sqrt{\frac{(1+k_1)t}{\mathrm{Pr}-1-k_1}}\right) + \exp\left(-My\sqrt{\frac{\mathrm{Pr}}{\mathrm{Pr}-1-k_1}}\right)\mathrm{erfc}\left(0.5y\sqrt{\frac{\mathrm{Pr}}{(1+k_1)t}}\right. \\
&\left.\left.-M\sqrt{\frac{(1+k_1)t}{\mathrm{Pr}-1-k_1}}\right)\right] + \frac{\mathrm{Gm}F_1}{M^2}\left[0.5\exp\left(\frac{M^2t}{\mathrm{Sc}-1}\right)\left\{exp\left(yM\sqrt{\frac{\mathrm{Sc}}{\mathrm{Sc}-1}}\right)\mathrm{erfc}\left(0.5y\sqrt{\frac{\mathrm{Sc}}{t}} +\right.\right.\right. \\
&\left.\left.\left.M\sqrt{\frac{t}{\mathrm{Sc}-1}}\right) + \exp\left(-yM\sqrt{\frac{\mathrm{Sc}}{\mathrm{Sc}-1}}\right)\mathrm{erfc}\left(0.5y\sqrt{\frac{\mathrm{Sc}}{t}} - M\sqrt{\frac{t}{\mathrm{Sc}-1}}\right)\right\}\right]. \qquad (4.2)
\end{aligned}
$$

4.3. **For vertical convection.** When $\alpha \to \frac{\pi}{2}, \sin\alpha \to 1, \cos\alpha \to 0$

$$
\begin{aligned}
&u(y,t) = \frac{1}{2}\left[\mathrm{e}^{My}\mathrm{erfc}\left(\frac{0.5y}{\sqrt{t}} + M\sqrt{t}\right) + \mathrm{e}^{-My}\mathrm{erfc}\left(\frac{y}{2\sqrt{t}} - M\sqrt{t}\right)\right] \\
&\left[1 - \frac{(\mathrm{Gr}+\mathrm{Gm})}{M^2}\right] + \frac{0.5\mathrm{Gr}}{M^2}\exp\left(\frac{M^2(1+k_1)t}{\mathrm{Pr}-1-k_1}\right) \\
&\left[\exp\left(My\sqrt{\frac{\mathrm{Pr}}{\mathrm{Pr}-1-k_1}}\right)\mathrm{erfc}\left(\frac{0.5y}{\sqrt{t}} + M\sqrt{\frac{\mathrm{Pr}}{\mathrm{Pr}-1-k_1}t}\right)\right. \\
&\left.+\exp\left(-My\sqrt{\frac{\mathrm{Pr}}{\mathrm{Pr}-1-k_1}}\right)\mathrm{erfc}\left(\frac{0.5y}{\sqrt{t}} - M\sqrt{\frac{\mathrm{Pr}}{\mathrm{Pr}-1-k_1}}\right)\right] \\
&+\frac{0.5\mathrm{Gm}}{M^2}\exp\left(\frac{M^2}{\mathrm{Sc}-1}t\right)\left[\exp\left(My\sqrt{\frac{\mathrm{Sc}}{\mathrm{Sc}-1}}\right)\mathrm{erfc}\left(\frac{0.5y}{\sqrt{t}} + M\sqrt{\frac{\mathrm{Sc}}{\mathrm{Sc}-1}t}\right)\right. \\
&\left.+\exp\left(-My\sqrt{\frac{\mathrm{Sc}}{\mathrm{Sc}-1}}\right)\mathrm{erfc}\left(\frac{0.5y}{\sqrt{t}} - M\sqrt{\frac{\mathrm{Sc}t}{\mathrm{Sc}-1}t}\right)\right] \\
&+\frac{1}{M^2}\left[\mathrm{Gmerfc}\left(0.5y\sqrt{\frac{\mathrm{Sc}}{t}}\right) + 0.5\mathrm{Gr}y\sqrt{\frac{\mathrm{Pr}}{(1+k_1)t}}\right] \\
&+\frac{1}{M^2}\left[\mathrm{Gmerfc}\left(0.5y\sqrt{\frac{\mathrm{Sc}}{t}}\right) + 0.5\mathrm{Gr}y\sqrt{\frac{\mathrm{Pr}}{(1+k_1)t}}\right] \\
&-\frac{0.5\mathrm{Gr}}{M^2}\exp\left(\frac{M^2(1+k_1)t}{\mathrm{Pr}-1-k_1} + \exp\left(My\sqrt{\frac{\mathrm{Pr}}{\mathrm{Pr}-1-k_1}}\right)\right) \\
&\mathrm{erfc}\left[\left(0.5y\sqrt{\frac{\mathrm{Pr}}{1+k_1t}} + M\sqrt{\frac{(1+k_1)t}{\mathrm{Pr}-1-k_1}}\right)\right. \\
&\left.+\exp\left(-My\sqrt{\frac{\mathrm{Pr}}{\mathrm{Pr}-1-k_1}}\right)\mathrm{erfc}\left(.5y\sqrt{\frac{\mathrm{Pr}}{(1+k_1)t}} - M\sqrt{\frac{(1+k_1)t}{\mathrm{Pr}-1-k_1}}\right)\right] \\
&\frac{\mathrm{Gm}}{M^2}\left[0.5\exp\left(\frac{M^2t}{\mathrm{Sc}-1}\right)\left\{\exp\left(yM\sqrt{\frac{\mathrm{Sc}}{\mathrm{Sc}-1}}\right)\mathrm{erfc}\left(0.5y\sqrt{\frac{\mathrm{Sc}}{t}} +\right.\right.\right. \\
&\left.\left.\left.M\sqrt{\frac{t}{\mathrm{Sc}-1}}\right) + \exp\left(-yM\sqrt{\frac{\mathrm{Sc}}{\mathrm{Sc}-1}}\right)\mathrm{erfc}\left(0.5y\sqrt{\frac{\mathrm{Sc}}{t}} - M\sqrt{\frac{t}{\mathrm{Sc}-1}}\right)\right\}\right]
\end{aligned} \tag{4.3}
$$

5. Results and discussion

Figures (2-8) shows the relationship between velocity distribution and various pertinent parameters. It is explored that the effect of square root of Hartmann number M over the velocity profile with the special coordinate is in orthogonal direction to the surface of solid boundary. In a dimensionless form, the hydromagnetic terms are in linear drag force terms in which as value of M is directly proportional to the strength of magnetic field. Accordingly, the velocity profile values are intensely reduced while increasing the value of M. It is also noted that as M increases, then the velocity profiles decline to zero gradually for smaller distances from the surface of the plate. The velocity profiles are increased mostly further from the plate with an increase in Grashof number Gr as shown in Figure (3). The drag forces oppose the flow in the perpendicular directions in the presence of inconstant mass diffusion. In Figure (4), the velocity profile is increased when the Grashof number values Gm are increased. The absolute values of velocity profile decrease with the increase of Schmidt number Sc, Prandtl number Pr, and along with it the molecular diffusivity of the chemical species decreases as shown in Figure (5) and (6). It is physically explored that Pr defines the proportion between momentum and thermal diffusivity along with controlling the thickness between thermal boundary layers and momentum. As, viscous forces govern the thermal diffusivity therefore the velocity of the fluid decreases when they are increased. For the fixed inclination $\alpha = \pi/3$, the velocity distribution is also decreased for various values of Hartmann number (M) as shown in the Figure (7). Figure (8) shows the relation between velocity distribution and plate inclinations (α). The velocity is minimized for $\alpha = 0$, (horizontal plate) and the effect of gravitational acceleration is negated. A gradual decay happen from rigid boundary to fluid streem. Velocity reverse its direction (decreases) also from the boundary and for less inclination angles. By gradually increasing the orientation of the plate from ($\alpha = \pi/6$, $\alpha = \pi/4$, $\alpha = \pi/3$) in vertical ($\alpha = \pi/2$) direction, the flow is accelerated. Finally, the effects of gravitational acceleration g are maximized and back flow is rejected. The velocity profiles relate to free convection but significant values of magnetic field strength ($M = 5$). Figures (9) and (10) show the relationship of temperature distribution profile (θ) for different values of Boltzmann Rosseland radiation convection parameter k_1 and time t. As the value of (Pr < 1), then the diffusion of heat is faster than momentum in the system. As $k_1 > 1$, then it is thermal radiation and for $k_1 < 1$ corresponds to thermal conduction. If $k_1 = 1$ then both radiation and conduction of heat transfer modes will equally contribute to the system. An addition in the values of k_1 which physically represents a thermal radiation flux. Temperature is obviously increased in the regime with an increase in time.

It is also seen from Figure (11-12) the concentration profiles of fluid are increased by increase of Schmidt number Sc. Further, under the constant physical parameters, the mass transfer will decrease as the values of Sc increases and similarly as the values of Sc is increased the concentration profiles are decreased. Furthermore, it is also observed that the distribution of concentration is increased by addition of time. The computational results of frictionless shear stress, concentration of the plate, and wall temperature are shown in tables with various variational parameters. Table 1 shows the relation between the constant square root of Hartmann number M and various values of plate inclination α. It also shows that by increasing the plate orientation, the frictional shear stress is increased and the square root of Hartmann number M remains constant and in contrast for any fixed value of the plate orientation, the shear stress is decreased. Table 2 shows that by increasing various values of inclination α under constant Grashof number, the value of frictional shear stress is increased and it also accelerates the speed of the flow. If the orientation of the plate is fixed then the magnitude of shear stress must be increased with the increase of Grashof number. In all cases, it gives back flow because the values are negative.

Table 3 shows that the value of frictional shear stress is increased as orientation of the plate is increased under constant Grashof number (Gm). Table 4 shows the influence of time when the Bolzmann-Rosseland number k_1 remains constant or varies for plate temperature gradient ($\frac{d\theta}{dy}\Big| y \rightarrow 0$. When the value of the Bolzmann-Rosseland number k_1 remains fixed with increasing the time span, the magnitude

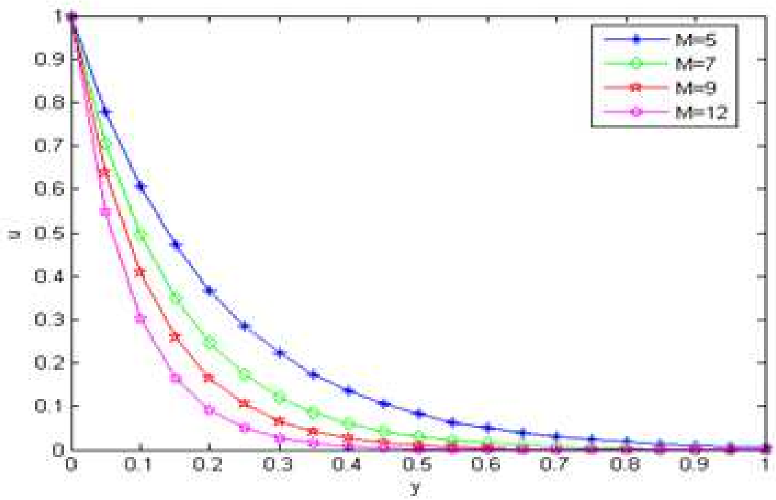

FIGURE 2. Distribution of velocity for different square root values of the Hartmann number (M) for $\alpha = \frac{\pi}{4}$, Gr=5, Gm=5, Sc=2, F=1, $k_1 = 1$, t=1, Pr=3

of temperature gradient is constantly reduced. But in contrast the value of heat transfer gradient is decreased when the time is fix and the value of k_1 is increased. Finally, in Table 5, the influence of time and Schmidt number Sc on concentration gradient of plate is presented. As the time is increased under fixed value of Sc then the concentration gradient of the plate is constantly reduced. Quite the reverse, for fixed time, with the increase in Sc, gradually decreases the magnitude of the concentration gradient of the plate.

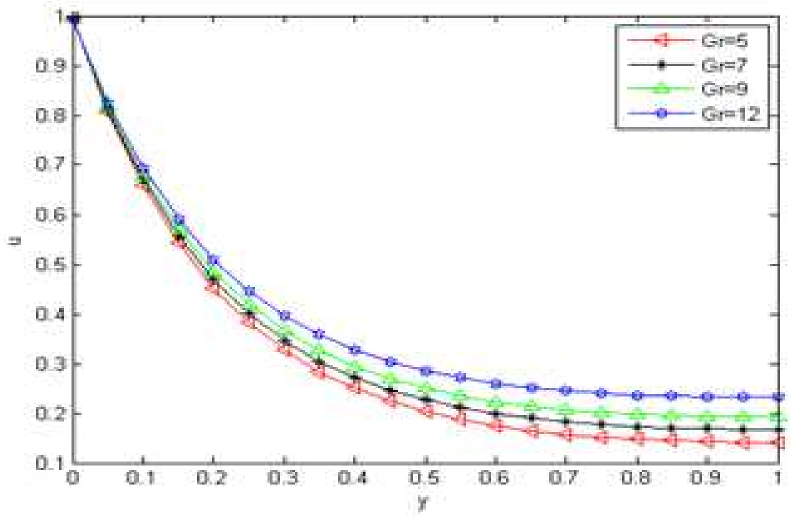

FIGURE 3. velocity distribution for diffirent Grashof numbers (Gr) for $\alpha = \frac{\pi}{3}$, M=5, Gm=5, Sc=2, F=1, $k_1 = 1$, t=1, Pr=3

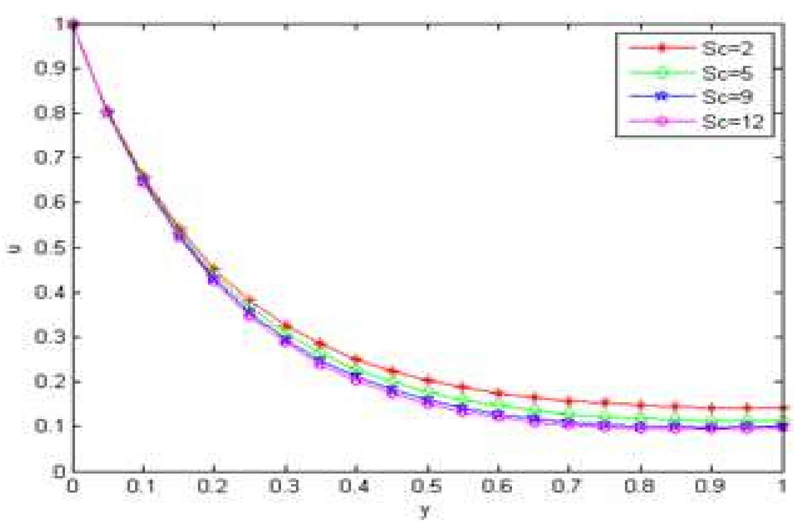

FIGURE 4. Distribution of velocity for different Grashof numbers (Gm) for $\alpha = \frac{\pi}{3}$, Gr=5, M=5, Sc=2, F=1, $k_1 = 1$, t=1, Pr=3

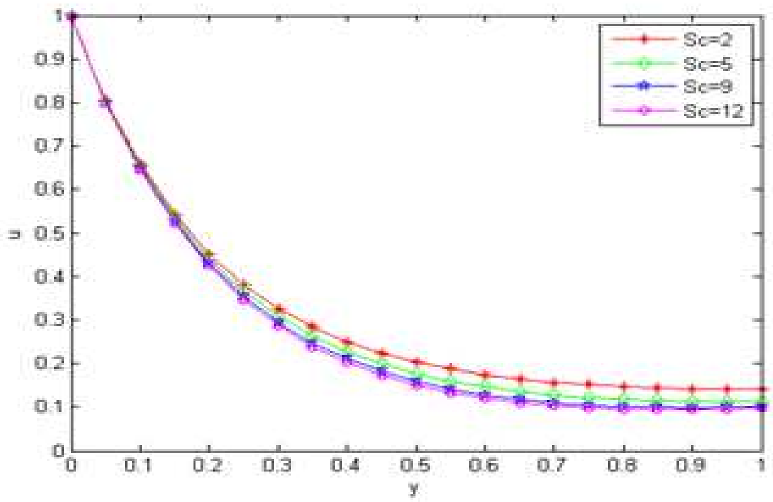

FIGURE 5. Distribution of velocity for various values of Schmidt number (Sc) for $\alpha = \frac{\pi}{3}$, M=5, Gr=5, Gm=5, F=1, $k_1 = 1$, t=1.

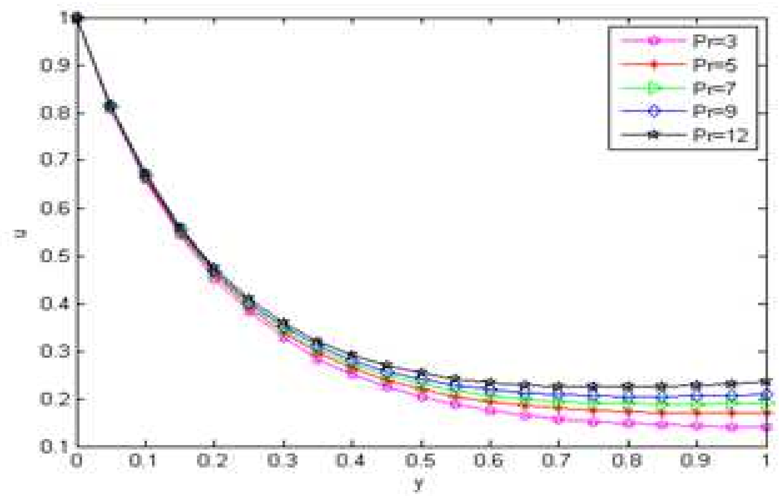

FIGURE 6. Distribution of velocity for different values of (Pr), for $\alpha = \frac{\pi}{3}$, Gr=5, Gm=5, Sc=2, F=1, k_1=1, t=1, M=5

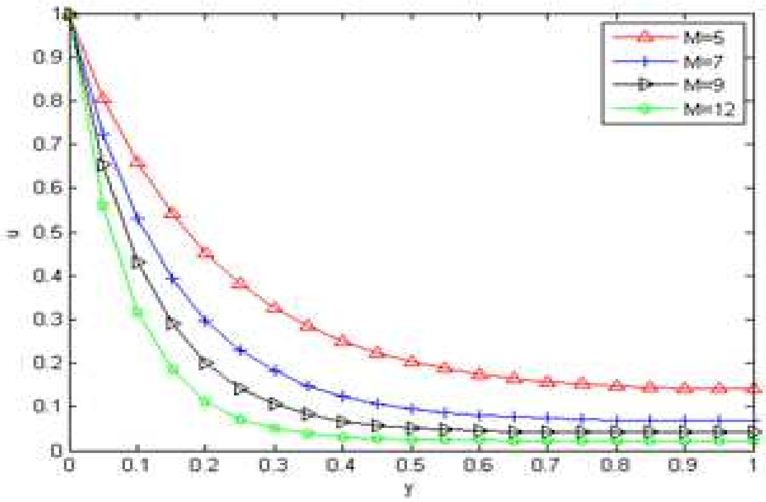

FIGURE 7. Distribution of velocity for different square root values of the Hartmann number (M) for $\alpha = \frac{\pi}{3}$, Gr=5, Gm=5, Sc=2, F=1, $k_1 = 1$, t=1, Pr=3

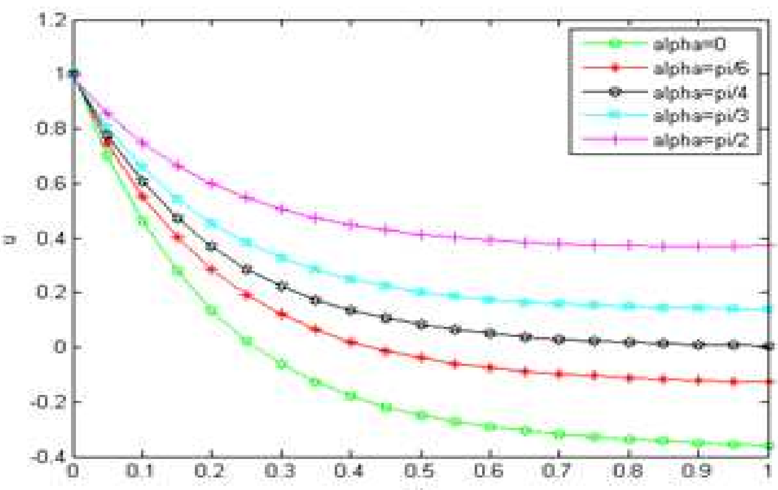

FIGURE 8. Distribution of velocity for various inclinations of plate for Gr=5, Gm=5, Sc=2, F=1, Pr=3, k_1=1, $t = 1$,

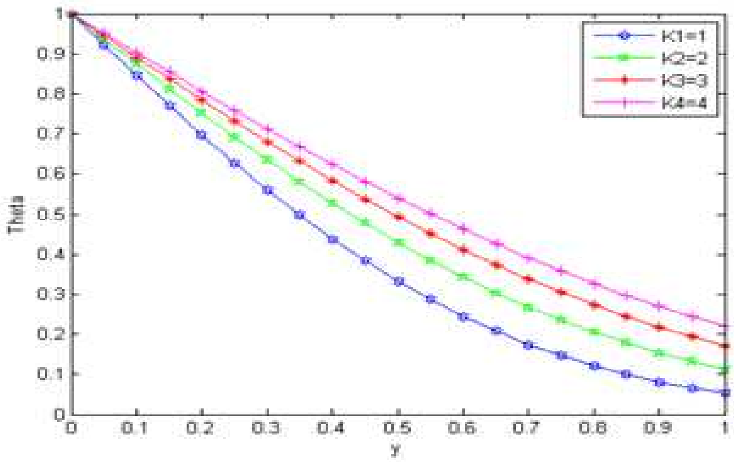

FIGURE 9. Distribution of velocity for various inclinations of plate

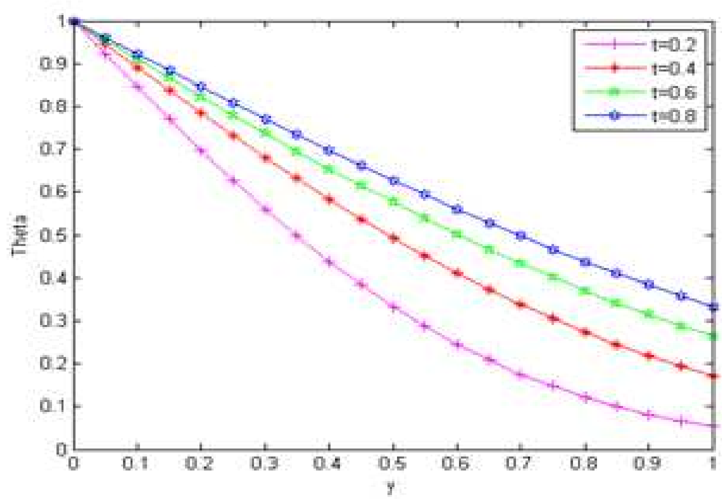

FIGURE 10. Distribution of temperature for different times, for Pr

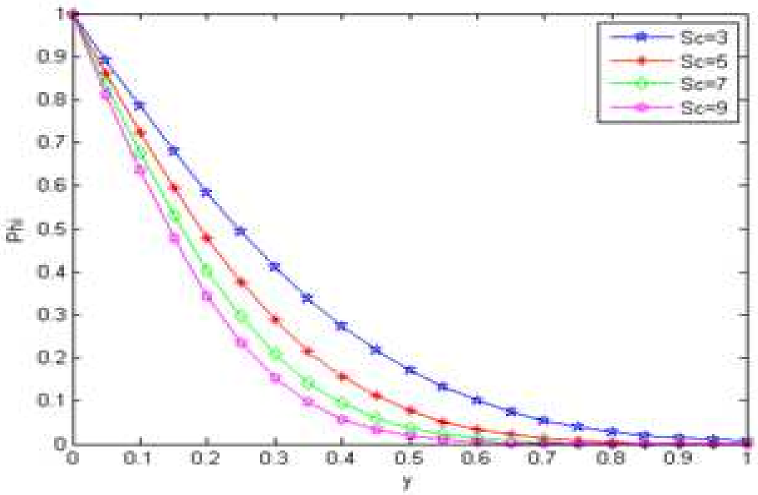

FIGURE 11. Concentration distribution with different values of parameter (Sc) at t=0.2

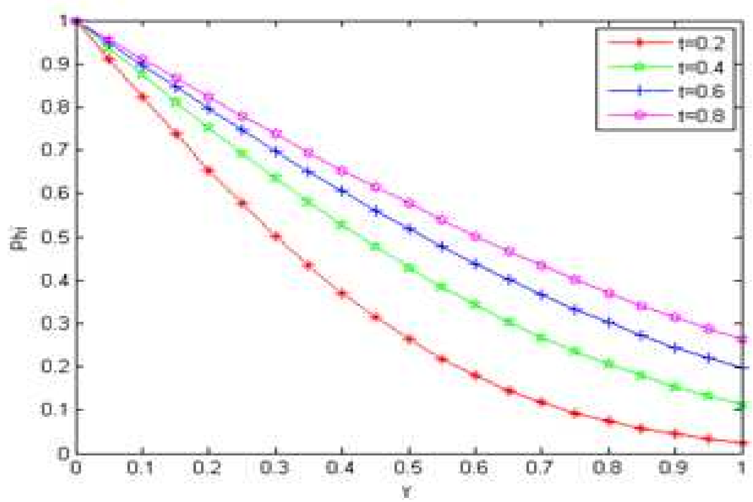

FIGURE 12. Concentration distribution for different time for Sc=2

Table: 1 Concentration distribution for different values of time for Sc=2

$M \rightarrow$ / $\alpha \downarrow$	5	7	9	12
0	-6.9301545	-8.3918727	-10.088637	-12.820587
Pi/6	-5.7064856	-7.5094608	-9.3984689	-12.300356
Pi/4	-5.00000	-7.00000	-9.00000	-12.00000
Pi/3	-4.2935144	-6.4905392	-8.6015311	-11.699644
Pi/2	-3.0698455	-5.6081273	-7.9113626	-11.179413

Table: 2 Fractional stress $\frac{du}{dy}_{(}y \rightarrow 0)$ and F_1=1, Pr=3, Gm=5, k_1=1, t=1, Sc=2 with various plate inclinations and thermal Grashof number

$Gr \rightarrow$ / $\alpha \downarrow$	5	7	9	12
0	-6.9301545	-7.3158505	-7.7015465	-8.2800906
Pi/6	-5.7064856	-5.8476601	-5.9888347	-6.2005965
Pi/4	-5.00000	-5.00000	-5.00000	-5.00000
Pi/3	-4.2935144	-4.1523399	-4.0111653	-3.7994035
Pi/2	-3.0698455	-2.6841495	-2.2984535	-1.7199094

Table: 3 Fractional stress $\frac{du}{dy}_{(y \rightarrow 0)}$ and F_1=1, Pr=3, k_1=1, t=1, Gr=5, Sc=2 with various plate inclinations and mass Grashof number

$Gm \rightarrow$ / $\alpha \downarrow$	5	7	9	12
0	-6.9301545	-7.3165203	-7.7028861	-8.2824348
Pi/6	-5.7064856	-5.8479053	-5.989325	-6.2014545
Pi/4	-5.00000	-5.00000	-5.00000	-5.00000
Pi/3	-4.2935144	-4.1520947	-4.010675	-3.7985455
Pi/2	-3.0698455	-2.6834797	-2.2971139	-1.7175652

Table: 4 Temperature gradient $\frac{d\theta}{dy}_{(y\to 0)}$ for different times t, Pr=0.71, and Boltzmann Rosseland constant k_1

$t \rightarrow$ / $k_1 \downarrow$	0.2	0.3	0.4	0.5
0.5	-0.867948	-0.708676	-0.613732	-0.548938
1	-0.751665	-0.613732	-0.531507	-0.475395
2	-0.613732	-0.50111	-0.433974	-0.388158
3	-0.531507	-0.433974	-0.375832	-0.336155
4	-0.475395	-0.388158	-0.336155	-0.300666

Table: 5 Concentration gradient $\frac{d\phi}{dy}_{(y\to 0)}$ for different times and Sc

$t \rightarrow$ / Sc $\downarrow$	0.2	0.3	0.4	0.5
0.71	-1.06301	-0.867948	-0.751665	-0.672309
0.73	-1.07788	-0.880088	-0.762178	-0.681713
0.75	-1.09255	-0.892062	-0.772548	-0.690988
0.77	-1.10702	-0.903878	-0.782781	-0.700141
0.79	-1.1213	-0.915541	-0.792882	-0.709175

6. Conclusions

Exact solutions of unsteady hydromagnetic fluid flow of viscous, optically thick, and incompressible electrically conducting liquid along an inclined plane with the transmission of heat and mass are systematically analyzed. Laplace transform method is used to obtain analytical solutions. The impact of the related parameters are also observed showing their significance on the results. The following results are obtained from this research, fluid velocity increases with the decrease of Schmidt number Sc and square root of Hartman number M and is increased for Grashof numbers Gr, Gm. Moreover, the absolute value of velocity distribution of the fluid decreases by an increase in the Prandtl number Pr, as defines the proportion between momentum and thermal diffusivity. The velocity of the fluid decreases by increasing the viscous forces because they govern the thermal diffusivity. The temperature distribution increases by an increase of time and radiational parameter. The fluid concentrations are increased by the decrease in Schmidt number along with the addition of time span.

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