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NEW SOLITARY WAVE STRUCTURES TO TIME FRACTIONAL BIOLOGICAL POPULATION MODEL

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ABSTRACT. Nonlinear space -time-fractional models perform an important task in revealing the internal devices of complex phenomena in numerous areas of the real world. This article examined time-fractional biological population model and gained some new solitary wave structures through the modified (G'/G)-expansion method. Among these results, a few solutions are obtained for the initial time. Finally, we concluded that the examined approach in raise, notable in showing numerous solitary wave structures of various nonlinear space -time-fractional models following in biology, physics and engineering as well.

1. INTRODUCTION

Nonlinear space -time-fractional models have been excited about the observation of numerous scientists in different areas. Everybody can display diverse natural phenomena to nonlinear space -time-fractional models. Furthermore, they define the dynamics of these phenomena and discover the physical application of these nonlinear space -time-fractional models. Several scientists have been attempted to receive distinct schemes that capable of performing the solitary wave structures of these nonlinear space -time-fractional models. Many effective methods have been used to solve nonlinear space -time-fractional models, these methods include rational (G'/G)-expansion method [1], the improved fractional riccati expansion method [2], He's variational iteration method [3], the homotopy analysis method [4], homotopy perturbation method [5, 6], the fractional reduced differential transform method [7], Lie symmetry analysis [8], modified generalized Taylor fractional series method [9], the first integral method [10], modified exp-function method [19], variable separation method [12], Chebyshev collocation method [13], method of separation variables [14], (G'/G)-expansion scheme [15, 16, 17, 18, 19, 20], generalized exponential rational function method [21], finite series Jacobi elliptic cosine function ansatz [22], shifted Jacobi spectral collocation method [23], modified auxiliary equation method [24], new generalized exponential rational function method [25], generalized unified method [26], the generalized exponential function [27], general bilinear form [28], reproducing kernel Hilbert space method [29], residual power series method [30], the exp$-\phi(\xi)$-expansion

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method [31, 32, 33], variation of parameters method [34], the traditional homotopy perturbation method [35], the improved $\tan(\phi(\xi)/2)$-expansion method [36, 37], the sumudu homotopy perturbation method [38], the sine-Gordon expansion method [39], riccati-bernoulli sub-ODE method [40], reproducing kernel method [41], the extended trial equation method [42], the variation of (G'/G)-expansion method [43], the improved (G'/G)-expansion method [44], the novel generalized (G'/G)-expansion method [45] and many more.

The paper applied the modified $(\frac{G\prime}{G})$-expansion method [46, 47] to derive the different type of solitary wave structures for time fractional biological population model [1, 2]. The time fractional biological population model as follows:

$$D_t^{\alpha}W - D_x^2W - D_y^2W - S_1(W^2 - S_2) = 0, \tag{1.1}$$

here W describes the population density, S_1, S_2 are free parameters and $s_1(W^2 - s_2)$ expresses the population supply owing to births and deaths. A biological population model is a mathematical model that helps us to comprehend the dynamical process of population changes and contributes important predictions. The nature that extends from simple to dynamic is complete of intercommunications. Most of the earths processes affect human life. Methods in population modeling have significantly improved our knowledge of biology and the natural life. A population model that is implemented to the investigation of population dynamics is a variety of mathematical model which produces us including a stable comprehension of how intricate communications and methods task. This investigation proposes to acquire novel solitary wave structures to the considered model via the modified $(\frac{G\prime}{G})$-expansion method.

2. Fractional derivative

Suppose that $g : x \to g(x)$. It's denotes a continuous but not significantly differentiable function. The fractional derivatives of order α are defined through the expression [48] as the following:

$$D_x^{\alpha} = \begin{cases} \frac{1}{\Gamma(-\alpha)} \int_0^x (x-\xi)^{-\alpha-1}[g(\xi) - g(0)]d\xi, \alpha < 0, \\ \frac{1}{\Gamma(-\alpha)} \frac{d}{dx} \int_0^x (x-\xi)^{-\alpha}[g(\xi) - g(0)]d\xi, 0 < \alpha < 1, \\ (g^{(m)}(x))^{\alpha - m}, m \leq \alpha \leq m+1, m \geq 1. \end{cases} \tag{2.1}$$

The Mittag-Leffler function including two parameters is explained which as follows [49]:

$$H_{\alpha,\beta}(x) = \sum_{i=0}^{\infty} \frac{x^i}{\Gamma(\alpha i + \beta)}, Re(\alpha) > 0, \beta, x \in C. \tag{2.2}$$

Some notable characteristics for the fractional derivative are given below, respectively:

(1) $D_x^{\alpha} x^{\gamma} = \frac{\Gamma(1+\gamma)}{\Gamma(1+\gamma-\alpha)} x^{\gamma-\alpha}, \gamma > 0.$
(2) $D_x^{\alpha}(cg(x)) = cD_x^{\alpha}(g(x)).$
(3) $D_x^{\alpha}(ag(x) + bh(x)) = aD_x^{\alpha}g(x) + bD_x^{\alpha}h(x).$

3. The fractional complex transformation

Suppose that the nonlinear fractional ODE:

$$R(W, D_t^{\alpha}W, D_x^{\beta}W, D_t^{\alpha}D_t^{\alpha}W, D_t^{\alpha}D_x^{\beta}W, D_x^{\beta}D_x^{\beta}W, \dots) = 0, \tag{3.1}$$

where $0 < \alpha \leq 0$, $0 < \beta \leq 0$, $D_t^{\alpha}W$ and $D_x^{\beta}W$ are the fractional derivatives of W with respect to t and x.

We study the equation (3.1) through $W = W(x,t) = W(\xi), \xi = \frac{kx^{\beta}}{\Gamma(1+\beta)} + \frac{\lambda t^{\beta}}{\Gamma(1+\alpha)}$. Hence, the equation (3.1) is converted to the following partially differential equation:

$$S(W, \frac{\partial W}{\partial \xi}, \frac{\partial^2 W}{\partial \xi^2}, \frac{\partial^3 W}{\partial \xi^3}, \dots) = 0. \tag{3.2}$$

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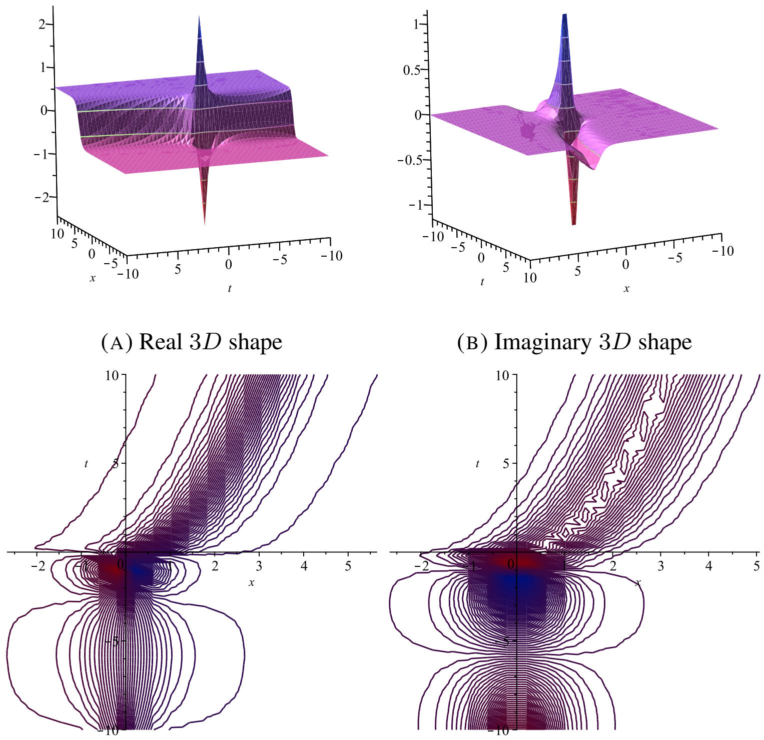
4. Solitary wave solutions for the time fractional biological population model



(A) Real $3D$ shape (B) Imaginary $3D$ shape

(C) Real contour plot (D) Imaginary contour plot

FIGURE 1. The three-dimensional and contour shape of the solution in $W_1(x, y, t)$ for $y = 0$, $\alpha = 0.5$, $d = 1$, $A = 1$, $B = 0$, $C = 4$, $E = 1$ $f_1 = -0.25$, $f_2 = -0.50$ and $f_3 = -0.75$.

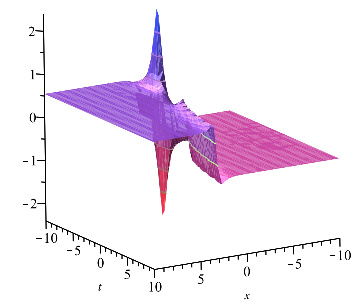
Let us consider the equation (1.1).

Making $W(x, y, t) = W(\eta)$, $\eta = ax + iay - \frac{bt^{\alpha}}{\alpha}$, $i^2 = -1$ into the equation (1.1), we derive the following equation:

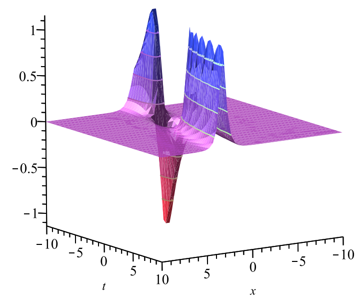
$$bW' - S_1(W^2 - S_2) = 0. \tag{4.1}$$

In accordance with the rule of the modified $(\frac{G\prime}{G})$-expansion method [46], equation (1.1) gives:

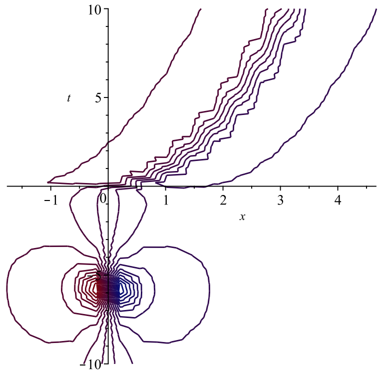
$$U(\xi) = A_1 F(\xi) + A_0 + A_{-1} F^{-1}(\xi), \tag{4.2}$$



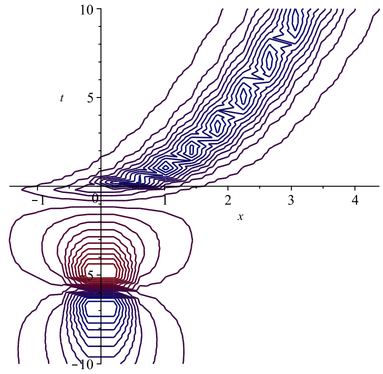
(A) Real $3D$ shape



(B) Imaginary $3D$ shape



(C) Real contour plot



(D) Imaginary contour plot

FIGURE 2. The three-dimensional and contour shape of the solution in $W_2(x, y, t)$ for $y = 0$, $\alpha = 0.5$, $d = 1$, $A = 1$, $B = 0$, $C = 4$, $E = 1$ $f_1 = -0.25$, $f_2 = -0.50$ and $f_3 = -0.75$.

where the coefficients A_0, A_1 and A_{-1} are constants. By equation (4.2) and equation (1.1) and then equating each coefficients of F^i to zeros, we get:

- The first set:

$$b = \frac{2S_1S_2}{(\pm\sqrt{S_2(\lambda^2 - 4\mu)})}, A_0 = 0, A_1 = 0, A_{-1} = \frac{1}{2}(\pm\sqrt{S_2(\lambda^2 - 4\mu)}).$$

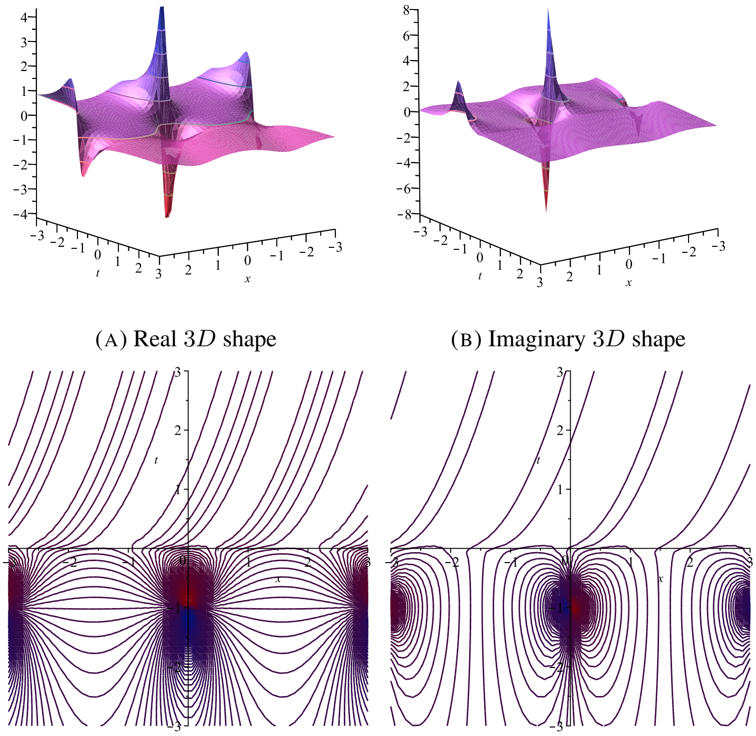
Using the values of the first set, from equation (4.2) and equation (1.1), we have:

$$W_1(x, y, t) = \frac{1}{2}(\pm\sqrt{S_2(\lambda^2 - 4\mu)}) \times coth\{\frac{\sqrt{\lambda^2 - 4\mu}}{2}(ax + iay - \frac{bt^\alpha}{\alpha})\}.$$

$$W_2(x, y, t) = \frac{1}{2}(\pm\sqrt{S_2(\lambda^2 - 4\mu)}) \times tanh\{\frac{\sqrt{\lambda^2 - 4\mu}}{2}(ax + iay - \frac{bt^\alpha}{\alpha})\}.$$

$$W_3(x, y, t) = \frac{1}{2}(\pm\sqrt{S_2(\lambda^2 - 4\mu)}) \times (ax + iay - \frac{bt^\alpha}{\alpha}).$$

$$W_4(x, y, t) = \frac{1}{2}(\pm\sqrt{S_2(\lambda^2 - 4\mu)}) \times cot\{\frac{\sqrt{4\mu - \lambda^2}}{2}(ax + iay - \frac{bt^\alpha}{\alpha})\}.$$



(A) Real $3D$ shape
(B) Imaginary $3D$ shape
(C) Real contour plot
(D) Imaginary contour plot

FIGURE 3. The three-dimensional and contour shape of the solution in $W_3(x, y, t)$ for $y = 0$, $\alpha = 0.5$, $d = 1$, $A = 1$, $B = 0$, $C = 4$, $E = 1$ $f_1 = -0.25$, $f_2 = -0.50$ and $f_3 = -0.75$.

$$W_5(x, y, t) = \frac{1}{2}(\pm\sqrt{S_2(\lambda^2 - 4\mu)}) \times tan\{\frac{\sqrt{4\mu - \lambda^2}}{2}(ax + iay - \frac{bt^\alpha}{\alpha})\}.$$

- The second set:

$$b = -S_1, A_0 = 0, A_1 = -\frac{4S_2}{\lambda^2 - 4\mu}, A_{-1} = 0.$$

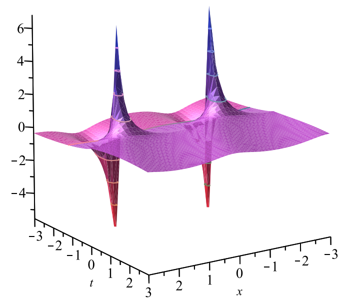
Similarly, we get:

$$W_6(x, y, t) = -\frac{2S_2}{\sqrt{\lambda^2 - 4\mu}} \times tanh\{\frac{\sqrt{\lambda^2 - 4\mu}}{2}(ax + iay - \frac{bt^\alpha}{\alpha})\}.$$

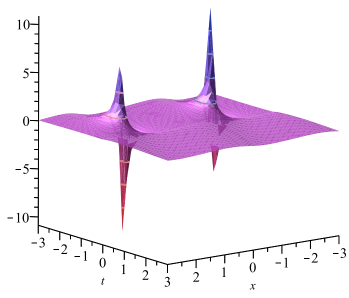
$$W_7(x, y, t) = -\frac{2S_2}{\sqrt{\lambda^2 - 4\mu}} \times coth\{\frac{\sqrt{\lambda^2 - 4\mu}}{2}(ax + iay - \frac{bt^\alpha}{\alpha})\}.$$

$$W_8(x, y, t) = -\frac{4S_2}{\lambda^2 - 4\mu} \times (ax + iay - \frac{bt^\alpha}{\alpha}).$$

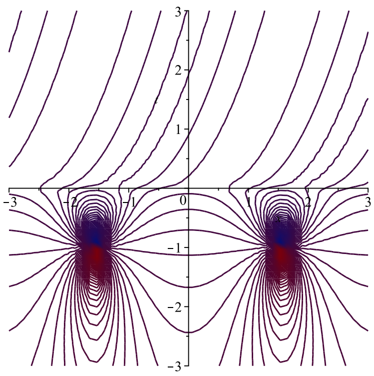
$$W_9(x, y, t) = -\frac{2S_2}{\sqrt{4\mu - \lambda^2}} \times tan\{\frac{\sqrt{4\mu - \lambda^2}}{2}(ax + iay - \frac{bt^\alpha}{\alpha})\}.$$



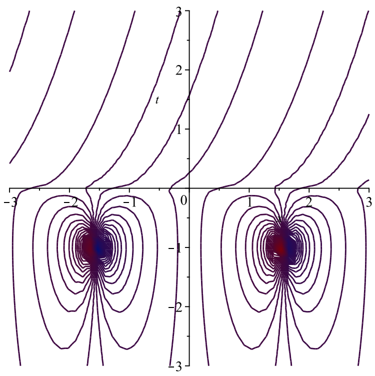
(A) Real $3D$ shape



(B) Imaginary $3D$ shape



(C) Real contour plot



(D) Imaginary contour plot

FIGURE 4. The three-dimensional and contour shape of the solution in $W_4(x,y,t)$ for $y=0$, $\alpha=0.5$, $d=1$, $A=1$, $B=0$, $C=4$, $E=1$ $f_1=-0.25$, $f_2=-0.50$ and $f_3=-0.75$.

$$W_{10}(x,y,t)=-\frac{2S_2}{\sqrt{4\mu-\lambda^2}}\times cot\{\frac{\sqrt{4\mu-\lambda^2}}{2}(ax+iay-\frac{bt^{\alpha}}{\alpha})\}.$$

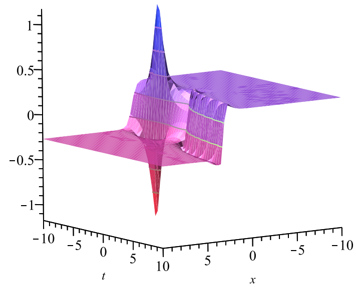
- The third set:

$$b = -S_1, A_0=0, A_1=-\frac{1}{3}\frac{-\lambda^2+4S_2+4\mu}{\lambda^2-4\mu}, A_{-1}=-\frac{1}{4}\lambda^2+\mu).$$

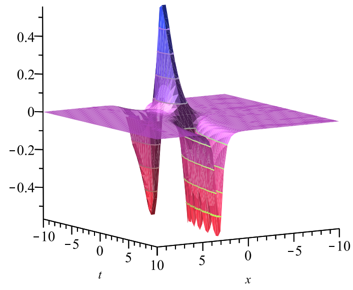
Similarly, we find:

$$W_{11}(x,y,t)=-\frac{1}{6}\frac{-\lambda^2+4S_2+4\mu}{\sqrt{\lambda^2-4\mu}}\times tanh\{\frac{\sqrt{\lambda^2-4\mu}}{2}(ax+iay-\frac{bt^{\alpha}}{\alpha})\}$$
$$-\frac{\sqrt{\lambda^2-4\mu}}{2}\times coth\{\frac{\sqrt{\lambda^2-4\mu}}{2}(ax+iay-\frac{bt^{\alpha}}{\alpha})\}.$$

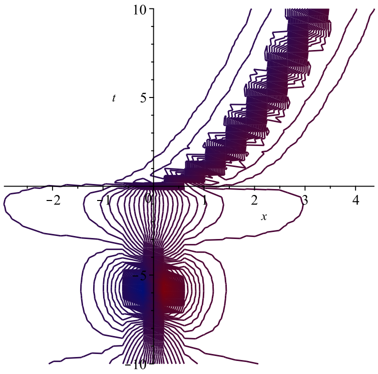
$$W_{12}(x,y,t)=-\frac{1}{6}\frac{-\lambda^2+4S_2+4\mu}{\sqrt{\lambda^2-4\mu}}\times coth\{\frac{\sqrt{\lambda^2-4\mu}}{2}(ax+iay-\frac{bt^{\alpha}}{\alpha})\}$$
$$-\frac{\sqrt{\lambda^2-4\mu}}{2}\times tanh\{\frac{\sqrt{\lambda^2-4\mu}}{2}(ax+iay-\frac{bt^{\alpha}}{\alpha})\}.$$



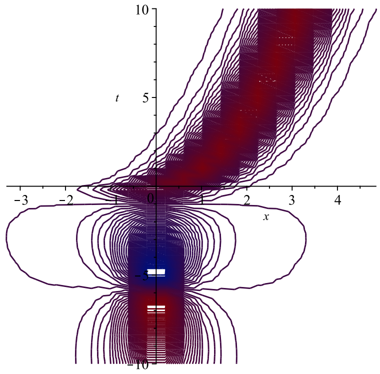
(A) Real $3D$ shape



(B) Imaginary $3D$ shape



(C) Real contour plot



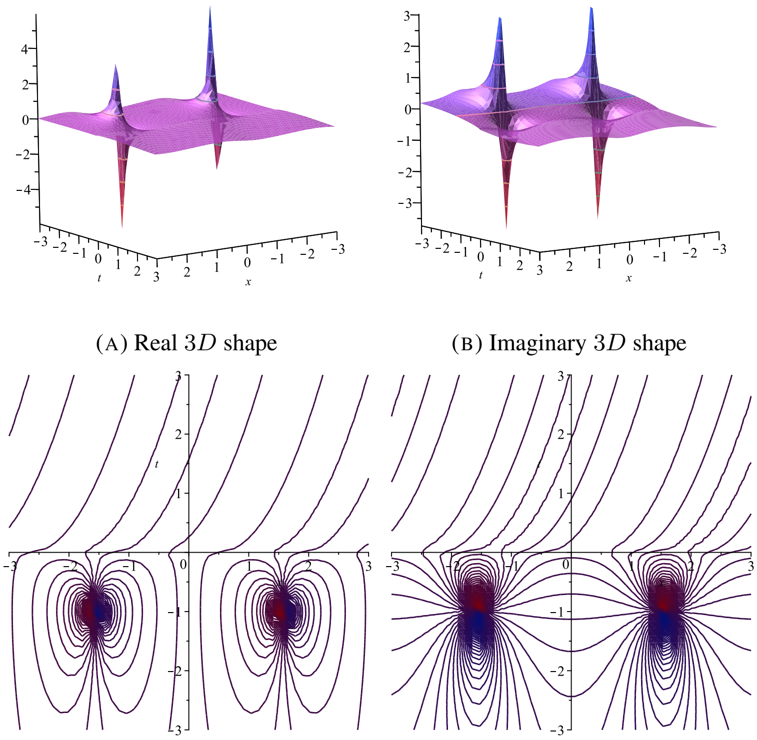
(D) Imaginary contour plot

FIGURE 5. The three-dimensional and contour shape of the solution in $W_6(x,y,t)$ for $y=0$, $\alpha=0.5$, $d=1$, $A=1$, $B=0$, $C=4$, $E=1$ $f_1=-0.25$, $f_2=-0.50$ and $f_3=-0.75$.

$$W_{13}(x,y,t) = -\frac{1}{3}\frac{-\lambda^2+4S_2+4\mu}{\lambda^2-4\mu} \times \frac{1}{(ax+iay-\frac{bt^\alpha}{\alpha})} \\ -\frac{1}{4}\lambda^2+\mu) \times (ax+iay-\frac{bt^\alpha}{\alpha}).$$

$$W_{14}(x,y,t) = \frac{1}{6}\frac{-\lambda^2+4S_2+4\mu}{\sqrt{4\mu-\lambda^2}} \times tan\{\frac{\sqrt{4\mu-\lambda^2}}{2}(ax+iay-\frac{bt^\alpha}{\alpha})\} \\ +\frac{\sqrt{4\mu}-\lambda^2}{2} \times cot\{\frac{\sqrt{4\mu-\lambda^2}}{2}(ax+iay-\frac{bt^\alpha}{\alpha})\}.$$

$$W_{15}(x,y,t) = \frac{1}{6}\frac{-\lambda^2+4S_2+4\mu}{\sqrt{4\mu-\lambda^2}} \times cot\{\frac{\sqrt{4\mu-\lambda^2}}{2}(ax+iay-\frac{bt^\alpha}{\alpha})\} \\ +\frac{\sqrt{4\mu-\lambda^2}}{2} \times tan\{\frac{\sqrt{4\mu-\lambda^2}}{2}(ax+iay-\frac{bt^\alpha}{\alpha})\}.$$



(A) Real $3D$ shape

(B) Imaginary $3D$ shape

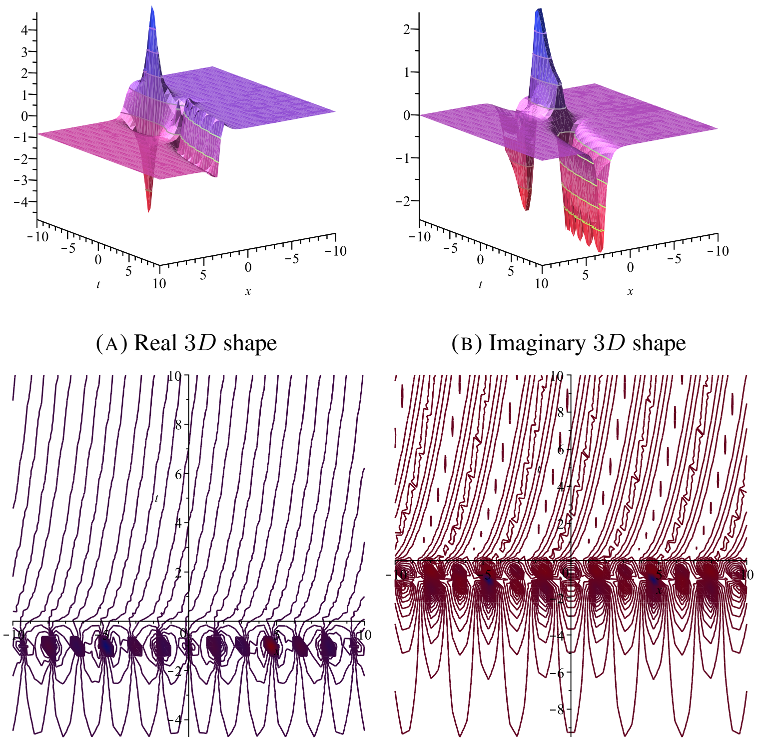
(C) Real contour plot

(D) Imaginary contour plot

FIGURE 6. The three-dimensional and contour shape of the solution in $W_8(x, y, t)$ for $y = 0$, $\alpha = 0.5$, $d = 1$, $A = 1$, $B = 0$, $C = 4$, $E = 1$ $f_1 = -0.25$, $f_2 = -0.50$ and $f_3 = -0.75$.

5. RESULTS AND DISCUSSIONS

In this paper, we apply the modified (G'/G)-expansion process [46] on the equation (1.1) and provide fifteen solitary wave structures. The obtained solitary wave structures to the equation (1.1) are new and general. Salam and Gumma [2] were applied to the improved fractional Riccati expansion method and constructed two traveling wave solutions. Salam and Gumma [2] only derived hyperbolic solutions but failed to achieve trigonometric and rational. Akbar et al. [1] was applied rational (G'/G)-expansion method on the same model and gained six closed-form traveling wave solutions. Comparison between three methods, the modified (G'/G)-expansion process is provided more solitary wave structures rather than the improved fractional Riccati expansion method and rational (G'/G)-expansion method. Finally, the newly method successfully implemented to derive new solitary wave structures to the equation (1.1). The graph is an important tool for information and to demonstrate the solutions to the problems lucidly. When making the computation in daily life, we need a fundamental knowledge of building the application of graphs. Accordingly, the graphical performances of few solutions are drawn in the Figures 1- 8, respectively. We expressed Figure 1- 8, respectively for few of the derived



(A) Real $3D$ shape (B) Imaginary $3D$ shape

(C) Real contour plot (D) Imaginary contour plot

FIGURE 7. The three-dimensional and contour shape of the solution in $W_{12}(x,y,t)$ for $y=0$, $\alpha=0.5$, $d=1$, $A=1$, $B=0$, $C=4$, $E=1$ $f_1=-0.25$, $f_2=-0.50$ and $f_3=-0.75$.

solutions to display more of properties for the recommended model. The representation of the examined process gives the accuracy and influence of this procedure and also the capacity for implementing various nonlinear wave models.

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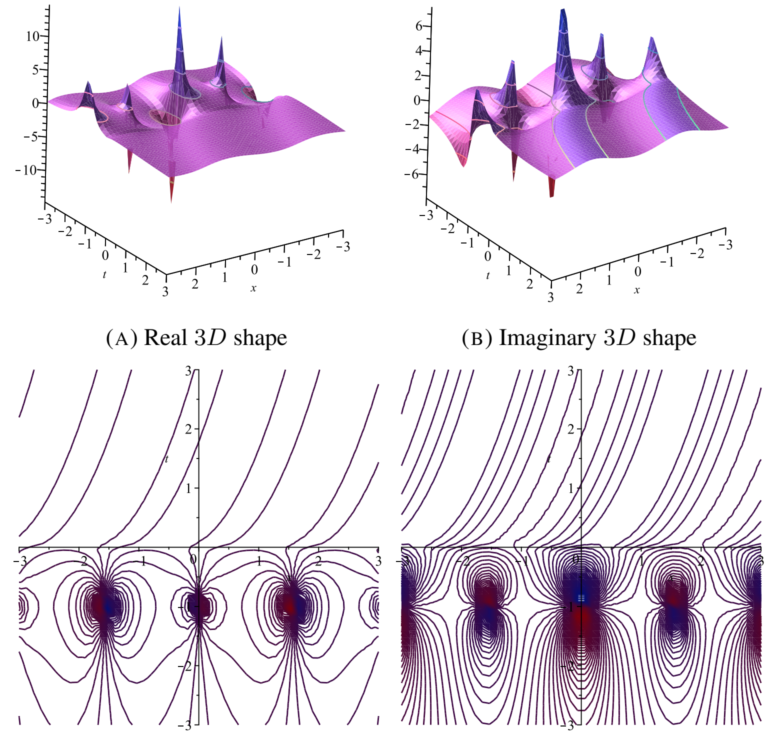

(A) Real $3D$ shape

(B) Imaginary $3D$ shape

(C) Real contour plot

(D) Imaginary contour plot

FIGURE 8. The three-dimensional and contour shape of the solution in $W_{13}(x, y, t)$ for $y = 0$, $\alpha = 0.5$, $d = 1$, $A = 1$, $B = 0$, $C = 4$, $E = 1$ $f_1 = -0.25$, $f_2 = -0.50$ and $f_3 = -0.75$.

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