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KANNAN FIXED POINT THEOREM IN $\mathcal{C}$-METRIC SPACE

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Abstract. The aim of this article is to define Kannan type contraction in the setting of controlled metric space and establish some contemporary fixed point theorems. These results extend several familiar results of literature . An example and certain consequences are presented to illustrate significance of established results.

1. Introduction and Preliminaries

In 1906, the self-evident evolution of a metric space was imperative accomplished by M. Frechet. The advantage of these spaces in the essential advancement of fixed point theory is gigantic. Banach's contraction principle [10] is one of the decisive outcomes of Functional Analysis and its applications, which provides that, if σ is a mapping from a complete metric space (Θ, ϖ) into itself and $\exists \vartheta \in [0, 1)$ such that

$$\varpi(\sigma\varrho, \sigma\varkappa) \leq \vartheta\varpi(\varrho, \varkappa)$$

for all $\varrho, \varkappa \in \Theta$, then σ has a unique fixed point in Θ. It is evident from the above contractive condition that σ is a continuous function. There are many generalization [1-23] of this result.

In 1968 Kannan [16] established a fixed point theorem for mapping satisfying

$$\varpi(\sigma\varrho, \sigma\varkappa) \leq \vartheta(\varpi(\varrho, \sigma\varrho) + \varpi(\varkappa, \sigma\varkappa))$$

for all $\varrho, \varkappa \in \Theta$, where $\vartheta \in [0, \frac{1}{2})$. The mapping used by Kannan may not always be continuous. Several researchers followed Kannan [16] paper using distinctive types of contractive conditions in metric spaces.

Inspired from this innovative idea, several mathematicians generalized and extended this conception in the current years as: probabilistic, pseudo, extended, partial , b-metric and controlled metric spaces. Presently there exists several literatures on these extensions of metric spaces.

Czerwik [11] defined the conception of b-metric space in this way.

Definition 1.1. *(see.* [11] *)Let $\Theta \neq \emptyset$ and $s \geq 1$. A function $\varpi : \Theta \times \Theta \to [0,\infty)$ is said to be a b-metric if these assertions hold:*

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($b1$) $\varpi(\varrho,\varkappa)=0$ if and only if $\varrho=\varkappa$;
($b2$) $\varpi(\varrho,\varkappa)=\varpi(\varkappa,\varrho)$ for all $\varrho,\varkappa\in\Theta$;
($b3$) $\varpi(\varrho,\varsigma)\leq s[\varpi(\varrho,\varkappa)+\varpi(\varkappa,\varsigma)]$ for all $\varrho,\varkappa,\varsigma\in\Theta$.
Then (Θ,ϖ) is said to be a b- metric space.

In 2017, Kamran et al. [17] commenced the abstraction of extended b-metric spaces.

Definition 1.2. *Let $\Theta\neq\emptyset$ and $F:\Theta\times\Theta\to[1,\infty)$. A function $\varpi:\Theta\times\Theta\to[0,\infty)$ is a controlled metric if for all $\varrho,\varkappa,\varsigma\in\Theta$, it satisfies*

(i) $\varpi(\varrho,\varkappa)=0$ if and only if $\varrho=\varkappa$;
(ii) $\varpi(\varrho,\varkappa)=\varpi(\varkappa,\varrho)$;
(iii) $\varpi(\varrho,\varkappa)\leq F(\varrho,\varkappa)[\varpi(\varrho,\varsigma)+\varpi(\varkappa,\varsigma)]$.
The pair (Θ,ϖ) is called an extended b-metric space .

Very recently, Mlaiki et al. [21] has introduced a new type of a generalized b-metric space.

Definition 1.3. [21]*Let $\Theta\neq\emptyset$ and $F:\Theta\times\Theta\to[1,\infty)$. A function $C:\Theta\times\Theta\to[0,\infty)$ is called a controlled metric type if for all $\varrho,\varkappa,\varsigma\in\Theta$, it satisfies*

(i) $\mathcal{C}(\varrho,\varkappa)=0$ if and only if $\varrho=\varkappa$;
(ii) $\mathcal{C}(\varrho,\varkappa)=C(\varkappa,\varrho)$;
(iii) $\mathcal{C}(\varrho,\varkappa)\leq F(\varrho,\varsigma)\mathcal{C}(\varrho,\varsigma)+F(\varkappa,\varsigma)\mathcal{C}(\varkappa,\varsigma)$.
Then $(\Theta,\mathcal{C},F)$ is called a controlled metric type space ($\mathcal{C}$-metric space).

Definition 1.4. [21]*Let $(\Theta,\mathcal{C},F)$ be a $\mathcal{C}$-metric space and $\{\varrho_r\}_{r\geq0}$ be a sequence in Θ.*

(1) $\{\varrho_r\}_{r\geq0}\longrightarrow\varrho\in\Theta$, if, $\forall\ \epsilon>0$, $\exists\ N=N(\epsilon)\in\mathbb{N}$ such that $\mathcal{C}(\varrho_r,\varrho)<\epsilon$, $\forall r\geq r$.
(2) $\{\varrho_r\}_{r\geq0}$ is Cauchy, if, $\forall\ \epsilon>0$, $\exists\ N=N(\epsilon)\in\mathbb{N}$ such that $\mathcal{C}(\varrho_t,\varrho_r)<\epsilon$, $\forall\ t,r\geq N$.
(3) If every Cauchy sequence in $\mathcal{C}$-metric space $(\Theta,\mathcal{C},F)$ is convergent, then it is said to be complete.

They proved Banach fixed point theorem in the setting of controlled metric spaces in this way.

Theorem 1.5. *Let (Θ,C,F) be a complete C-metric space and $\sigma:\Theta\to\Theta$ be such that*

$$\mathcal{C}(\sigma\varrho,\sigma\varkappa)\leq\vartheta(\mathcal{C}(\varrho,\varkappa))$$

for all $\varrho,\varkappa\in\Theta$, where $\vartheta\in(0,1)$. For $\varrho_0\in\Theta$, take $\varrho_r=\sigma^r\varrho_0$. Suppose that

$$\sup_{t\geq1}\lim_{i\to\infty}\frac{F(\varrho_{i+1},\varrho_{i+2})F(\varrho_{i+1},\varrho_t)}{F(\varrho_i,\varrho_{i+1})}<\frac{1}{\lambda}.$$

In addition, assume that, for every $\varrho\in\Theta$, we have $\lim_{r\to\infty}F(\varrho_r,\varrho)$ and $\lim_{r\to\infty}F(\varrho,\varrho_r)$ exist and are finite. Then, $\varrho^*=\sigma\varrho^*$.

In the present paper, we define Kannan type contractions in the setting of $\mathcal{C}$-metric space and obtain some original fixed point result. We also provide a non trivial example to illustrate significance of established results.

2. Main results

Theorem 2.1. *Let* (Θ, C, F) *be a complete* C*-metric space. Let* $\sigma : \Theta \to \Theta$ *such that*

$$\mathcal{C}(\sigma\varrho, \sigma\varkappa) \leq \vartheta(\mathcal{C}(\varrho, \sigma\varrho) + \mathcal{C}(\varkappa, \sigma\varkappa)) \tag{2.1}$$

for all $\varrho, \varkappa \in \Theta$*, where* $\vartheta \in (0, \frac{1}{2})$*. For* $\varrho_0 \in \Theta$*, take* $\varrho_r = \sigma^r \varrho_0$*. Assume that*

$$\sup_{t \geq 1} \lim_{i \to \infty} \frac{F(\varrho_{i+1}, \varrho_{i+2}) F(\varrho_{i+1}, \varrho_t)}{F(\varrho_i, \varrho_{i+1})} < \frac{1}{\lambda}. \tag{2.2}$$

In addition, assume that, for every $\varrho \in \Theta$, we have $\lim_{r\to\infty} F(\varrho_r, \varrho)$ and $\lim_{r\to\infty} F(\varrho, \varrho_r)$ exist and are finite. Then $\exists\ \varrho^* \in \Theta$ such that $\varrho^* = \sigma\varrho^*$.

Proof. Let $\varrho_0 \in \Theta$. We generate $\{\varrho_r\}$ in Θ by $\varrho_{r+1} = \sigma\varrho_r$, $\forall\ r \in \mathbb{N}$. Obviously, if $\exists$ $r_0 \in \mathbb{N}$ for which $\varrho_{r_0+1} = \varrho_{r_0}$, then $\sigma\varrho_{r_0} = \varrho_{r_0}$ and the proof is finished. Thus, we assume that $\varrho_{r+1} \neq \varrho_r$ for every $r \in \mathbb{N}$. Hence by (2.1), we have

$$\begin{aligned} \mathcal{C}(\varrho_r, \varrho_{r+1}) &= \mathcal{C}(\sigma\varrho_{r-1}, \sigma\varrho_r) \\ &\leq \vartheta(\mathcal{C}(\varrho_{r-1}, \sigma\varrho_{r-1}) + \mathcal{C}(\varrho_r, \sigma\varrho_r)) \\ &= \vartheta(\mathcal{C}(\varrho_{r-1}, \varrho_r) + \mathcal{C}(\varrho_r, \varrho_{r+1})) \end{aligned}$$

which implies

$$\mathcal{C}(\varrho_r, \varrho_{r+1}) \leq (\frac{\vartheta}{1-\vartheta})\mathcal{C}(\varrho_{r-1}, \varrho_r) = \lambda\mathcal{C}(\varrho_{r-1}, \varrho_r).$$

Similarly,

$$\begin{aligned} \mathcal{C}(\varrho_{r-1}, \varrho_r) &= \mathcal{C}(\sigma\varrho_{r-2}, \sigma\varrho_{r-1}) \\ &\leq \vartheta(\mathcal{C}(\varrho_{r-2}, \sigma\varrho_{r-2}) + \mathcal{C}(\varrho_{r-1}, \sigma\varrho_{r-1})) \\ &= \vartheta(\mathcal{C}(\varrho_{r-2}, \varrho_{r-1}) + \mathcal{C}(\varrho_{r-1}, \varrho_r)) \end{aligned}$$

which implies that

$$\mathcal{C}(\varrho_{r-1}, \varrho_r) \leq (\frac{\vartheta}{1-\vartheta})\mathcal{C}(\varrho_{r-2}, \varrho_{r-1}) = \lambda\mathcal{C}(\varrho_{r-2}, \varrho_{r-1}).$$

Continuing in this way, we have

$$\mathcal{C}(\varrho_r, \varrho_{r+1}) \leq \lambda\mathcal{C}(\varrho_{r-1}, \varrho_r) \leq \lambda^2\mathcal{C}(\varrho_{r-2}, \varrho_{r-1}) \leq ... \leq \lambda^r\mathcal{C}(\varrho_0, \varrho_1). \tag{2.3}$$

For all $r, t \in \mathbb{N}(r < t)$, we have

$$\begin{aligned}
\mathcal{C}(\varrho_r, \varrho_t) &\leq F(\varrho_r, \varrho_{r+1})\mathcal{C}(\varrho_r, \varrho_{r+1}) + F(\varrho_{r+1}, \varrho_t)\mathcal{C}(\varrho_{r+1}, \varrho_t) \\
&\leq F(\varrho_r, \varrho_{r+1})\mathcal{C}(\varrho_r, \varrho_{r+1}) + F(\varrho_{r+1}, \varrho_t)F(\varrho_{r+1}, \varrho_{r+2})\mathcal{C}(\varrho_{r+1}, \varrho_{r+2}) \\
&\quad + F(\varrho_{r+1}, \varrho_t)F(\varrho_{r+2}, \varrho_t)\mathcal{C}(\varrho_{r+2}, \varrho_t) \\
&\leq F(\varrho_r, \varrho_{r+1})\mathcal{C}(\varrho_r, \varrho_{r+1}) + F(\varrho_{r+1}, \varrho_t)F(\varrho_{r+1}, \varrho_{r+2})\mathcal{C}(\varrho_{r+1}, \varrho_{r+2}) \\
&\quad + F(\varrho_{r+1}, \varrho_t)F(\varrho_{r+2}, \varrho_t)F(\varrho_{r+2}, \varrho_{r+3})\mathcal{C}(\varrho_{r+2}, \varrho_{r+3}) \\
&\quad + F(\varrho_{r+1}, \varrho_t)F(\varrho_{r+2}, \varrho_t)F(\varrho_{r+3}, \varrho_t)\mathcal{C}(\varrho_{r+3}, \varrho_t) \\
&\leq \cdots \\
&\leq F(\varrho_r, \varrho_{r+1})\mathcal{C}(\varrho_r, \varrho_{r+1}) + \sum_{i=r+1}^{t-2} \left(\prod_{j=r+1}^{i} F(\varrho_j, \varrho_t) \right) F(\varrho_i, \varrho_{i+1})\mathcal{C}(\varrho_i, \varrho_{i+1}) \\
&\quad + \prod_{i=r+1}^{t-1} F(\varrho_i, \varrho_t)\mathcal{C}(\varrho_{t-1}, \varrho_t)
\end{aligned}$$

which further implies that

$$\begin{aligned}
\mathcal{C}(\varrho_r, \varrho_t) &\leq F(\varrho_r, \varrho_{r+1})\mathcal{C}(\varrho_r, \varrho_{r+1}) + \sum_{i=r+1}^{t-2} \left(\prod_{j=r+1}^{i} F(\varrho_j, \varrho_t) \right) F(\varrho_i, \varrho_{i+1})\mathcal{C}(\varrho_i, \varrho_{i+1}) \\
&\quad + \left(\prod_{i=r+1}^{t-1} F(\varrho_i, \varrho_t) \right) F(\varrho_{t-1}, \varrho_t)\mathcal{C}(\varrho_{t-1}, \varrho_t) \\
&\leq F(\varrho_r, \varrho_{r+1})\lambda^r\mathcal{C}(\varrho_0, \varrho_1) + \sum_{i=r+1}^{t-2} \left(\prod_{j=r+1}^{i} F(\varrho_j, \varrho_t) \right) F(\varrho_i, \varrho_{i+1})k^i\mathcal{C}(\varrho_0, \varrho_1) \\
&\quad + \left(\prod_{i=r+1}^{t-1} F(\varrho_i, \varrho_t) \right) F(\varrho_{t-1}, \varrho_t)\lambda^{t-1}\mathcal{C}(\varrho_0, \varrho_1)
\end{aligned}$$

$$= F(\varrho_r, \varrho_{r+1})\lambda^r\mathcal{C}(\varrho_0, \varrho_1) + \sum_{i=r+1}^{t-1} \left(\prod_{j=r+1}^{i} F(\varrho_j, \varrho_t) \right) F(\varrho_i, \varrho_{i+1})\lambda^i\mathcal{C}(\varrho_0, \varrho_1). \quad (2.4)$$

Let

$$S_l = \sum_{i=0}^{l} \left(\prod_{j=0}^{i} F(\varrho_j, \varrho_t) \right) F(\varrho_i, \varrho_{i+1})\lambda^i\mathcal{C}(\varrho_0, \varrho_1).$$

From (2.4), we get

$$\mathcal{C}(\varrho_r, \varrho_t) \leq \mathcal{C}(\varrho_0, \varrho_1)[\lambda^r F(\varrho_r, \varrho_{r+1}) + (S_{t-1} - S_r)] \quad (2.5)$$

Above, we make use of that $F(\varrho, \varkappa) \geq 1$, and by using ratio test, $\lim_{r\to\infty} S_r$ exists. Thus $\{S_r\}$ is Cauchy. Eventually, if we apply the limit as $r, t \to \infty$ in the inequality (2.5) , we conclude that

$$\lim_{r,t\to\infty} \mathcal{C}(\varrho_r, \varrho_t) = 0. \quad (2.6)$$

Thus, $\{\varrho_r\}$ is Cauchy in (Θ, C). So $\exists\ \varrho^* \in \Theta$ such that

$$\lim_{r\to\infty} \mathcal{C}(\varrho_r, \varrho^*) = 0 \quad (2.7)$$

that is $\varrho_r \to \varrho^*$ as $r \to \infty$. Now we show that $\varrho^* = \sigma\varrho^*$. By (2.1) and condition (iii), we get

$$\begin{aligned}\mathcal{C}(\varrho^*, \sigma\varrho^*) &\leq F(\varrho^*, \varrho_{r+1})\mathcal{C}(\varrho^*, \varrho_{r+1}) + F(\varrho_{r+1}, \sigma\varrho^*)\mathcal{C}(\varrho_{r+1}, \sigma\varrho^*) \\ &= F(\varrho^*, \varrho_{r+1})\mathcal{C}(\varrho^*, \varrho_{r+1}) + F(\varrho_{r+1}, \sigma\varrho^*)\mathcal{C}(\sigma\varrho_r, \sigma\varrho^*) \\ &\leq F(\varrho^*, \varrho_{r+1})\mathcal{C}(\varrho^*, \varrho_{r+1}) + F(\varrho_{r+1}, \sigma\varrho^*)[\vartheta(\mathcal{C}(\varrho_r, \sigma\varrho_r) + \mathcal{C}(\varrho^*, \sigma\varrho^*))] \\ &= F(\varrho^*, \varrho_{r+1})\mathcal{C}(\varrho^*, \varrho_{r+1}) + F(\varrho_{r+1}, \sigma\varrho^*)[\vartheta(\mathcal{C}(\varrho_r, \varrho_{r+1}) + \mathcal{C}(\varrho^*, \sigma\varrho^*))].\end{aligned}$$

Taking the limit as $r \to \infty$ and using (2.7), we get a contradiction to $\mathcal{C}(\varrho^*, \sigma\varrho^*) > 0$. Thus $\mathcal{C}(\varrho^*, \sigma\varrho^*) = 0$ This yields that $\varrho^* = \sigma\varrho^*$. This completes the proof. □

Example 2.2. *Let* $\Theta = \{0, 1, 2\}$ *Define* $F : \Theta\times\Theta \to [1, \infty)$ *and* $\mathcal{C} : \Theta\times\Theta \to [1, \infty)$ *as* $F(\varrho, \varkappa) = 1 + \varrho\varkappa$ *and*

$$\begin{aligned}\mathcal{C}(2, 2) &= \mathcal{C}(0, 0) = \mathcal{C}(1, 1) = 0 \\ \mathcal{C}(2, 0) &= \mathcal{C}(0, 2) = 5, \mathcal{C}(0, 1) = \mathcal{C}(0, 1) = 10 \\ \mathcal{C}(1, 2) &= \mathcal{C}(2, 1) = 30.\end{aligned}$$

Now, define

$$\sigma : \Theta \to \Theta$$

by

$$\sigma\varrho = \begin{cases} 0, & \text{if } \varrho \in \{0, 2\} \\ 2, & \text{if } \varrho = 1. \end{cases}$$

and choose $\vartheta = \frac{1}{3}$*. Now we discuss various cases to prove the assumptions of our main result.*

Case 01: If $\varrho = 0, \varkappa = 1$, we have

$$\mathcal{C}(\sigma 0, \sigma 1) = \mathcal{C}(0, 2) = 5 < \frac{1}{3}(30) = \frac{1}{3}\mathcal{C}(1, 2) = \frac{1}{3}(\mathcal{C}(0, 0) + \mathcal{C}(1, \sigma 1)).$$

Case 02: If $\varrho = 0, \varkappa = 2$, we have

$$\mathcal{C}(\sigma 0, \sigma 2) = \mathcal{C}(0, 0) = 0 < \frac{1}{3}(5) = \frac{1}{3}\mathcal{C}(2, 0) = \frac{1}{3}(\mathcal{C}(0, \sigma 0) + \mathcal{C}(2, \sigma 2)).$$

Case 03: If $\varrho = 1, \varkappa = 2$, we have

$$\mathcal{C}(\sigma 1, \sigma 2) = \mathcal{C}(2, 0) = 5 < \frac{1}{3}(35) = \frac{1}{3}(30 + 5) = \frac{1}{3}(\mathcal{C}(1, \sigma 1) + \mathcal{C}(2, \sigma 2)).$$

Case 04: If $\varrho = \varkappa = 0, \varrho = \varkappa = 1, \varrho = \varkappa = 2$, we have

$$\mathcal{C}(\sigma\varrho, \sigma\varkappa) = 0.$$

Consequently,

$$\mathcal{C}(\sigma\varrho, \sigma\varkappa) \leq \vartheta(\mathcal{C}(\varrho, \sigma\varrho) + \mathcal{C}(\varkappa, \sigma\varkappa))$$

for all $\varrho, \varkappa \in \Theta$. Thus, all the assumptions of above result are satisfied and $0 = \sigma 0$.

We can establish variety of results as special cases of our main result 2.1.

Special Cases:

Corollary 2.3. *Let* $\sigma : (\Theta, \varpi_e) \to (\Theta, \varpi_e)$ *be such that*

$$\varpi_e(\sigma\varrho, \sigma\varkappa) \leq \vartheta(\varpi_e(\varrho, \sigma\varrho) + \varpi_e(\varkappa, \sigma\varkappa))$$

for all $\varrho, \varkappa \in \Theta$*, where* $\vartheta \in (0, \frac{1}{2})$*. For* $\varrho_0 \in \Theta$*, take* $\varrho_r = \sigma^r\varrho_0$*. Assume that*

$$\sup_{t\geq 1}\lim_{i\to\infty}\frac{\digamma(\varrho_{i+1},\varrho_{i+2})\digamma(\varrho_{i+1},\varrho_t)}{\digamma(\varrho_i,\varrho_{i+1})}<\frac{1}{\lambda}.$$

In addition, assume that, for every $\varrho\in\Theta$, we have $\lim_{r\to\infty}\digamma(\varrho_r,\varrho)$ and $\lim_{r\to\infty}\digamma(\varrho,\varrho_r)$ exist and are finite. Then $\exists\ \varrho^*\in\Theta$ such that $\varrho^*=\sigma\varrho^*$.

Proof. Take $\digamma(\varrho,\varsigma)=\digamma(\varsigma,\varkappa)$ in above result 2.1. □

Corollary 2.4. *Let* $\sigma:(\Theta,\varpi_b)\to(\Theta,\varpi_b)$. *If* $\exists\ \vartheta\in(0,\frac{1}{2})$ *such that*

$$\varpi_b(\sigma\varrho,\sigma\varkappa)\leq\vartheta(\varpi_b(\varrho,\sigma\varrho)+\varpi_b(\varkappa,\sigma\varkappa))$$

for all $\varrho,\varkappa\in\Theta$. *Then* $\exists\ \varrho^*\in\Theta$ *such that* $\varrho^*=\sigma\varrho^*$.

Proof. Take $\digamma(\varrho,\varsigma)=\digamma(\varsigma,\varkappa)=s\geq 1$ in above result 2.1. □

Corollary 2.5. *Let* $\sigma:(\Theta,\varpi)\to(\Theta,\varpi)$. *If* $\exists\ \vartheta\in(0,\frac{1}{2})$ *such that*

$$\varpi(\sigma\varrho,\sigma\varkappa)\leq\vartheta(\varpi(\varrho,\sigma\varrho)+\varpi(\varkappa,\sigma\varkappa))$$

for all $\varrho,\varkappa\in\Theta$. *Then* $\exists\ \varrho^*\in\Theta$ *such that* $\varrho^*=\sigma\varrho^*$.

Proof. Take $\digamma(\varrho,\varsigma)=\digamma(\varsigma,\varkappa)=1$ in above Theorem 2.1. □

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