Journal of Inequalities and Special Functions
ISSN: 2217-4303, URL: http://www.ilirias.com
Volume 4 Issue 3(2013), Pages 29-37.

APPLICATION OF GROUP THEORY TO GENERATING RELATIONS FOR SPECIAL FUNCTIONS.

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Abstract. In this article, we have suggested a unified group theoretic method of obtaining a general class of generating relations involving various special functions from the existence of their partial quasi-bilateral (or bilinear) generating relations from the group theoretic point of view. The detail discussion of the method of obtaining generating function has been given in this paper and finally we obtain a theorem in connection with the unification of a class of generating relations involving some special functions. Furthermore, a good number of theorems and results on generating functions involving various special functions have been obtained in course of application of our main result (Theorem-1) obtained in the present investigation.

1. Introduction

In [1], partial quasi-bilateral generating function is defined as follows:

$$G(x,z,w) = \sum_{n=0}^{\infty} a_n \; p_{n+r}^{(\alpha)}(x) \; q_m^{(n+r)}(z) \; w^n,$$

where $p_{n+r}^{(\alpha)}(x)$ and $q_m^{(n+r)}(z)$ are two special functions of orders (n+r), m and of parameters α and (n+r).

In the present article, we have proved a general theorem in connection with the unification of a class of generating relations for various special functions from the existence of a partial quasi-bilateral generating function by using the concept of one-parameter group of continuous transformations. We have obtained the following theorem as the main result of our investigation.

Theorem-1: If

$$G(x,\; u,\; w) = \sum_{n=0}^{\infty} a_n \; p_{n+r}^{(\alpha)}(x) \; q_m^{(n+r)}(u) \; w^n, \tag{1.1}$$

2000 *Mathematics Subject Classification.* 33C45 .
Key words and phrases. Partial quasi-bilateral generating function, Laguerre polynomials, Bessel polynomials, Gegenbauer polynomials, Jacobi polynomials.

Submitted March 12, 2012. Published September 2, 2013.

then

$$\Omega_r^{'}(x,1,1)\ \Omega_r^{''}(u,1)\ \Big\{h_1(x,1,1)\Big\}^{\alpha}\ G\Big(g_1(x,1,1),\ g_2(u,1),\ wvh_2(u,1)k(x,1,1)\Big)$$
$$=\sum_{n=0}^{\infty}\sum_{p=0}^{\infty}\sum_{q=0}^{\infty} a_n\ v^n\ \frac{w^{p+q+n}}{p!\ q!}\ C_{1,r}\ C_{2,r}\ p_{n+r+p}^{(\alpha-p)}(x)\ q_m^{n+r+q}(u). \qquad (1.2)$$

Here, we would like to mention that in course of application of the result stated in Theorem-1, we have obtained the extensions of the quasi-bilinear generating functions involving Laguerre, Bessel, Gegenbauer and Jacobi polynomials. Furthermore, these quasi-bilinear generating relations, obtained in Cor.2 - Cor.6 as particular cases of Theorem-2 - Theorem-6, are the extensions of the corresponding bilateral generating functions found derived in [2,3,4,5,6].

2. Group theoretic Discussion

We first consider the partial quasi-bilateral generating relation:

$$G(x,\ u,\ w)=\sum_{n=0}^{\infty} a_n\ p_{n+r}^{(\alpha)}(x)\ q_m^{(n+r)}(u)\ w^n. \qquad (2.1)$$

Now replacing w by $wztv$ in (2.1) and then multiplying both sides by y^{α}, we get

$$y^{\alpha}\ G(x,\ u,\ wztv)=\sum_{n=0}^{\infty} a_n\ \Big(p_{n+r}^{(\alpha)}(x)\ y^{\alpha}z^n\Big)\ \Big(q_m^{(n+r)}(u)\ t^n\Big)\ \Big(vw\Big)^n. \qquad (2.2)$$

At first we suppose that for the special functions $p_{n+r}^{(\alpha)}(x)$ and $q_m^{(n+r)}(u)$, it is possible to find the following linear partial differential operators each of which generates one parameter group of continuous transformations:

$$\begin{cases} R_1=\xi_1(x,y,z)\frac{\partial}{\partial x}+\xi_2(x,y,z)\frac{\partial}{\partial y}+\xi_3(x,y,z)\frac{\partial}{\partial z}+\xi_0(x,y,z) \\ R_2=\eta_1(u,t)\frac{\partial}{\partial u}+\eta_2(u,t)\frac{\partial}{\partial t}+\eta_0(u,t) \end{cases}$$

such that

$$R_1\Big(p_{n+r}^{(\alpha)}(x)\ y^{\alpha}z^n\Big)=C_{n,\ r}^{'}\ p_{n+r+1}^{(\alpha-1)}(x)\ y^{\alpha-1}z^{n+1} \qquad (2.3)$$

and

$$R_2\Big(q_m^{(n+r)}(u)\ t^n\Big)=C_{n,\ r}^{''}\ q_m^{(n+r+1)}(u)\ t^{n+1}. \qquad (2.4)$$

We now assume that the groups generated by R_1 and R_2 are

$$e^{wR_1}\ f(x,\ y,\ z)=\Omega_r^{'}(x,y,z)\ f\Big(g_1(x,y,z),\ h_1(x,y,z),\ k(x,y,z)\Big) \qquad (2.5)$$

and

$$e^{wR_2}\ f(u,\ t) = \Omega_r^{''}(u,t)\ f\Big(g_2(u,t),\ h_2(u,t)\Big). \tag{2.6}$$

Operating $e^{wR_1}e^{wR_2}$ on both sides of (2.2) we get,

$$\begin{aligned} & e^{wR_1}e^{wR_2}\Big(y^{\alpha}\ G(x,\ u,\ wztv)\Big) \\ &= e^{wR_1}e^{wR_2}\sum_{n=0}^{\infty} a_n\ \Big(p_{n+r}^{(\alpha)}(x)\ y^{\alpha}z^n\Big)\ \Big(q_m^{(n+r)}(u)\ t^n\Big)\ \Big(vw\Big)^n. \end{aligned} \tag{2.7}$$

The left hand side of (2.7), with the help of (2.5) & (2.6), reduces to

$$\Omega_r^{'}(x,y,z)\ \Omega_r^{''}(u,t)\ \{h_1(x,y,z)\}^{\alpha}\ G\Big(g_1(x,y,z),\ g_2(u,t),\ wvh_2(u,t)k(x,y,t)\Big). \tag{2.8}$$

The right hand side of (2.7), with the help of (2.3) & (2.4), becomes

$$\begin{aligned} &\sum_{n=0}^{\infty}\sum_{p=0}^{\infty}\sum_{q=0}^{\infty} a_n\ v^n\ \frac{w^{p+q+n}}{p!\ q!}\ C_{n,\ r}^{'}\ C_{n+1,\ r}^{'}\ C_{n+2,\ r}^{'}\ .....C_{n+p-1,\ r}^{'}\ p_{n+p+r}^{(\alpha-p)}\ y^{(\alpha-p)} \\ &\qquad \times\ z^{(n+r)}\ C_{n,\ r}^{''}\ C_{n+1,\ r}^{''}\ C_{n+2,\ r}^{''}\ .....C_{n+q-1,\ r}^{''}\ q_m^{(n+q+r)}(u)\ t^{n+q}. \end{aligned} \tag{2.9}$$

Equating (2.8) & (2.9) and then putting y = z= t= 1, we get

$$\begin{aligned} & \Omega_r^{'}(x,1,1)\ \Omega_r^{''}(u,1)\ \Big(h_1(x,1,1)\Big)^{\alpha}\ G\Big(g_1(x,1,1),\ g_2(u,1),\ wvh_2(u,1)k(x,1,1)\Big) \\ &= \sum_{n=0}^{\infty}\sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\ a_n\ v^n\ \frac{w^{p+q+n}}{p!\ q!}\ C_{1,\ r}\ C_{2,\ r}\ p_{n+r+p}^{(\alpha-p)}(x)\ q_m^{n+r+q}(u), \end{aligned}$$

where $C_{1,r} = \prod_{i=0}^{p-1} C_{n+i,\ r}^{'}$ and $C_{2,\ r} = \prod_{i=0}^{q-1} C_{n+i,\ r}^{''}$.

Hence we get the following theorem:

Theorem-1: If

$$G(x,\ u,\ w) = \sum_{n=0}^{\infty} a_n\ p_{n+r}^{(\alpha)}(x)\ q_m^{(n+r)}(u)\ w^n,$$

then

$$\Omega_r^{'}(x,1,1)\ \Omega_r^{''}(u,1)\ \Big\{h_1(x,1,1)\Big\}^{\alpha}\ G\Big(g_1(x,1,1),\ g_2(u,1),\ wvh_2(u,1)k(x,1,1)\Big)$$

$$= \sum_{n=0}^{\infty}\sum_{p=0}^{\infty}\sum_{q=0}^{\infty} a_n \ v^n \ \frac{w^{p+q+n}}{p! \ q!} \ C_{1,r} \ C_{2,r} \ p_{n+r+p}^{(\alpha-p)}(x) \ q_m^{n+r+q}(u),$$

where $C_{1,\ r} = \prod_{i=0}^{p-1} C'_{n+i,\ r}$ and $C_{2,\ r} = \prod_{i=0}^{q-1} C''_{n+i,\ r}$, which does not seem to have appeared in the earlier investigations.

Cor 1: If we put r=0 in theorem-1, we immediately get the result in connection with the unification of quasi-bilateral generating relations involving various special functions found derived in [7].

3. Applications

3.1. **Laguerre Polynomial.** At first we take

$$p_{n+r}^{(\alpha)}(x) = L_{n+r}^{(\alpha)}(x), \ q_m^{(n+r)}(u) = L_m^{(n+r)}(u)$$

Then from [8,9], we get

$$\begin{array}{ll} R_1 = xy^{-1}z\frac{\partial}{\partial x} + z\frac{\partial}{\partial y} - xy^{-1}z & R_2 = t\frac{\partial}{\partial u} - t \\ C'_{n,r} = (r+n+1) & C''_{n,r} = (-1) \\ \Omega'_r(x,y,z) = exp(-wxy^{-1}z) & \Omega''_r(u,t) = exp(-wt) \\ g_1(x,y,z) = x(1+wy^{-1}z) & g_2(u,t) = u+wt \\ h_1(x,y,z) = y+wz & h_2(u,t) = t \\ k(x,y,z) = z & \end{array}.$$

Therefore, by the application of our theorem, we get the following result involving Laguerre polynomials.

Theorem-2: If

$$G(x, \ u, \ w) = \sum_{n=0}^{\infty} a_n \ L_{n+r}^{(\alpha)}(x) \ L_m^{(n+r)}(u) \ w^n,$$

then

$$exp\Big[-w(1+x)\Big] \ \Big(1+w\Big)^{\alpha} \ G\Big(x(1+w), \ u+w, \ uw\Big)$$

$$= \sum_{n=0}^{\infty}\sum_{p=0}^{\infty}\sum_{q=0}^{\infty} a_n \ v^n \ \frac{w^{p+q+n}}{p! \ q!} \ (r+n+1)_p \ (-1)^q \ L_{n+r+p}^{(\alpha-p)}(x) \ L_m^{(n+r+q)}(u),$$

which is found derived in [10].

Cor-2: If we put r=0, in the above theorem we get the result found derived in [7,11].

Special Case-1: If, in the above theorem, we put m=0 we get a novel extension of the bilateral generating function of Laguerre polynomials found derived in [12].

3.2. **Bessel Polynomials.** We now take

$$p_{n+r}^{(\alpha)}(x) = Y_{n+r}^{(\alpha)}(x)$$
$$q_m^{(n+r)}(u) = Y_m^{(n+r)}(u)$$

Then from [13,14], we notice that

$$R_1 = x^2y^{-1}z\frac{\partial}{\partial x} + xz\frac{\partial}{\partial y} + xy^{-1}z^2\frac{\partial}{\partial z} + (\beta + rx - x)y^{-1}z$$
$$R_2 = ut\frac{\partial}{\partial u} + t^2\frac{\partial}{\partial t} + (m + r - 1)t$$
$$C'_{n,r} = \beta \qquad C''_{n,r} = (m + n + r - 1)$$
$$\Omega'_r(x, y, z) = (1 - wxy^{-1}z)^{1-r}\ exp(\beta wy^{-1}z) \qquad \Omega''_r(u, t) = (1 - wt)^{1-m-r}$$
$$g_1(x, y, z) = \frac{x}{(1-wxy^{-1}z)} \qquad g_2(u, t) = \frac{u}{(1-wt)}$$
$$h_1(x, y, z) = \frac{y}{(1-wxy^{-1}z)} \qquad h_2(u, t) = \frac{t}{(1-wt)}$$
$$k(x, y, z) = \frac{z}{(1-wxy^{-1}z)}$$

Then, by the application of our theorem-1, we get the following result involving generalized Bessel polynomials.

Theorem-3: If

$$G(x,\ u,\ w) = \sum_{n=0}^{\infty} a_n\ Y_{n+r}^{(\alpha)}(x)\ Y_m^{(n+r)}(u)\ w^n,$$

then

$$exp\Big(\beta w\Big)\Big(1 - w\Big)^{1-m-r}\Big(1 - wx\Big)^{1-\alpha}\ G\Big(\frac{x}{1 - wx},\ \frac{u}{1 - w},\ \frac{wv}{(1 - wx)(1 - w)}\Big)$$
$$= \sum_{n=0}^{\infty}\sum_{p=0}^{\infty}\sum_{q=0}^{\infty} a_n\ v^n\ \frac{w^{p+q+n}}{p!\ q!}\ \beta^p\ (m + n + r - 1)_q\ Y_{n+r+p}^{(\alpha-p)}(x)\ Y_m^{(n+r+q)}(u)\ ,$$

which is found derived in [15].

Cor-3: If we put r=0 in the above theorem, we get the result found derived in [7]

Special Case-2: If, in the above theorem, we put m=0 we get the result found derived in [12].

3.3. **Gegenbauer polynomials.** We now take

$$p_{n+r}^{(\alpha)}(x) = C_{n+r}^{(\lambda)}(x)$$
$$q_m^{(n+r)}(u) = C_m^{(n+r)}(u)$$

then from [16,17]

$$R_1 = (x^2 - 1)y^{-1}z\frac{\partial}{\partial x} + 2xz\frac{\partial}{\partial y} - xy^{-1}z \qquad R_2 = ut\frac{\partial}{\partial u} + 2t^2\frac{\partial}{\partial t} + (m + 2r)t$$
$$C'_{n,r} = \frac{(n+r+2\lambda-1)(n+r+1)}{2(\lambda-1)} \qquad C''_{n,r} = 2(n + r)$$
$$\Omega'_r(x, y, z) = \{1 + 2wxy^{-1}z + (x^2 - 1)w^2y^{-2}z^2\}^{-\frac{1}{2}} \qquad \Omega''_r(u, t) = (1 - 2wt)^{-\frac{m}{2}-r}$$
$$g_1(x, y, z) = x + w(x^2 - 1)y^{-1}z \qquad g_2(u, t) = \frac{u}{\sqrt{1-2wt}}$$
$$h_1(x, y, z) = y\{1 + 2wxy^{-1}z + (x^2 - 1)w^2y^{-2}z^2\} \qquad h_2(u, t) = \frac{t}{\sqrt{1-2wt}}$$
$$k(x, y, z) = z$$

Then by the application of our theorem, we get the following result involving Gegenbauer polynomials.

Theorem-4: If

$$G(x,\ u,\ w) = \sum_{n=0}^{\infty} a_n\ C_{n+r}^{(\lambda)}(x)\ C_m^{(n+r)}(u)\ w^n,$$

then

$$\left\{1{+}2wx{+}(x^2{-}1)w^2\right\}^{-\frac{1}{2}+\lambda}\left(1{-}2w\right)^{-\frac{m}{2}-r}G\left(\ x{+}w(x^2{-}1),\ \frac{u}{\sqrt{1-2w}},\ \frac{wv}{1-zw}\ \right)$$

$$= \sum_{n=0}^{\infty}\sum_{p=0}^{\infty}\sum_{q=0}^{\infty} a_n\ v^n\ \frac{w^{p+q+n}}{p!\ q!}\ \frac{(-n-2\lambda-r+1)_p\ (n+r+1)_p}{2^p\ (1-\lambda)_p}\ 2^q\ (n{+}r)_q\ C_{n+r+p}^{(\lambda-p)}(x)\ C_m^{n+r+q}(u),$$

which is found derived in [18].

Cor-4: If we put r=0 in the above theorem, we get the result on quasi-bilateral generating relation found derived in [7].

Special Case-3: If, in the above corollary, we put m=0 we get the result found derived in [4].

3.4. **Jacobi Polynomials.** Now we take

$$p_{n+r}^{(\alpha)}(x) = P_{n+r}^{(k_1,\ \alpha)}(x)$$
$$q_m^{(n+r)}(u) = P_m^{(k_1,\ n+r)}(u)$$

Then from [13,19] we get

$$R_1 = (1-x^2)y^{-1}z\tfrac{\partial}{\partial x} + (1-x)z\tfrac{\partial}{\partial y} - (1+x)y^{-1}z^2\tfrac{\partial}{\partial z} - (1+k_1+r)(1+x)y^{-1}z$$
$$R_2 = (1-u)t\tfrac{\partial}{\partial u} - t^2\tfrac{\partial}{\partial t} - (1+n+k_1+m+r)t$$
$$C_{n,r}^{'} = -2(n+r+1) \qquad C_{n,r}^{''} = -(1+k_1+n+m+r)$$
$$\Omega_r^{'}(x,y,z) = \left\{1+w(1+x)y^{-1}z\right\}^{-1-k_1-r} \qquad \Omega_r^{''}(u,t) = (1+wt)^{-k_1-m-r-1}$$
$$g_1(x,y,z) = \tfrac{x+w(1+x)y^{-1}z}{1+w(1+x)y^{-1}z} \qquad g_2(u,t) = \tfrac{u+tw}{1+tw}$$
$$h_1(x,y,z) = \tfrac{y(1+2wy^{-1}z)}{1+w(1+x)y^{-1}z} \qquad h_2(u,t) = \tfrac{t}{1+tw}$$
$$k(x,y,z) = \tfrac{z}{1+w(1+x)y^{-1}z}.$$

Therefore by the application of our theorem, we get the following result on partial quasi-bilateral generating functions involving Jacobi polynomials.

Theorem-5: If

$$G(x,u,w) = \sum_{n=0}^{\infty} a_n\ P_{n+r}^{(k_1,\ \alpha)}(x)\ P_m^{(k_1,\ n+r)}(u)\ w^n,$$

then

$$\Big\{1+w(1+x)\Big\}^{-1-k_1-\alpha-r}\Big(1+w\Big)^{-1-k_1-m-r}\Big(1+2w\Big)^{\alpha}$$
$$\times G\Big(\frac{x+w(1+x)}{1+w(1+x)},\ \frac{u+w}{1+w},\ \frac{wv}{(1+w)\{1+w(1+x)\}}\Big)$$
$$=\sum_{n=0}^{\infty}\sum_{p=0}^{\infty}\sum_{q=0}^{\infty} a_n\, v^n\, \frac{w^{p+q+n}}{p!\ q!}\,(-2)^p\,(n{+}r{+}1)_p\,(-1)^q\,(1{+}k_1{+}m{+}n{+}r)_q\, P_{n+r+p}^{(k_1,\alpha-p)}(x)\, P_m^{(k_1,n+r+q)}(u),$$

which also does not seem to have appeared in the earlier works.

Cor-5: If we put r=0 in the above theorem, we get the result found derived in [7].

Special Case-4: If, in the above theorem, we put m=0, we get the result found derived in [12].

If in place of $R_1,\ R_2$ we take the following two operators from [8,19]:

$$R_1^{'}=(1-x^2)y^{-1}z\frac{\partial}{\partial x}-(1-x)z\frac{\partial}{\partial y}+(1-x)y^{-1}z^2+(1+\beta+r)(1-x)y^{-1}z$$
$$R_2^{'}=(1+u)t\frac{\partial}{\partial u}+t^2\frac{\partial}{\partial t}+(1+m+\beta+r)t$$

Then we have,

$$C_{n,r}^{'}=-2(n+r+1) \qquad C_{n,r}^{''}=(1+\beta+n+m+r)$$
$$\Omega_r^{'}(x,y,z)=\Big\{1+w(x-1)y^{-1}z\Big\}^{-1-\beta-r} \qquad \Omega_r^{''}(u,t)=(1-wt)^{-(1+m+\beta+r)}$$
$$g_1(x,y,z)=\frac{x-w(x-1)y^{-1}z}{1+w(x-1)y^{-1}z} \qquad g_2(u,t)=\frac{u+wt}{1-wt}$$
$$h_1(x,y,z)=\frac{y(1-2wy^{-1}z)}{1+w(x-1)y^{-1}z} \qquad h_2(u,t)=\frac{t}{1-wt}$$
$$k(x,y,z)=\frac{z}{1+w(x-1)y^{-1}z}$$

Then by the application of our theorem, we get the following analogous result on partial quasi-bilateral generating relation involving Jacobi polynomials:

Theorem-6: If

$$G(x,u,w)=\sum_{n=0}^{\infty} a_n\ P_{n+r}^{(\alpha,\ \beta)}(x)\ P_m^{(n+r,\ \beta)}(u)\ w^n$$

then

$$\Big\{1+w(x-1)\Big\}^{-(1+\beta+r)}\Big(1-w\Big)^{-(1+m+\beta+r)}\Big\{\frac{1-2w}{1+w(x-1)}\Big\}^{\alpha}$$
$$\times\ G\Big(\frac{x-w(x-1)}{1+w(x-1)},\ \frac{u+w}{1-w},\ \frac{wv}{(1-w)\{1+w(x-1)\}}\Big)$$
$$=\sum_{n=0}^{\infty}\sum_{p=0}^{\infty}\sum_{q=0}^{\infty} a_n\, v^n\, \frac{w^{p+q+n}}{p!\ q!}\,(-2)^p\,(n{+}r{+}1)_p\,(1{+}\beta{+}m{+}n{+}r)_q\, P_{n+r+p}^{(\alpha-p,\ \beta)}(x)\, P_m^{(n+r+q,\ \beta)}(u),$$

which is found derived in [20].

Cor-6: If we put r=0 in the above theorem, we get the result found derived in [7].

Special Case-5: If, in the above theorem, we put m=0, we get the result found derived in [12].

4. Conclusions:

From the above discussion, it is clear that one may apply theorem-1 in the case of other polynomials and functions existing in the field of special functions to obtain the partial quasi-bilinear (or bilateral) generating functions involving the special function(s) under consideration subject to the condition of construction of one parameter continuous transformations group for the said special function(s). Furthermore, one may observe that the main result obtained in this paper is the most general form of the extension of the quasi-bilateral generating function involving $p_n^{(\alpha)}(x)$, $q_m^{(n)}(u)$ from the existence of a partial quasi-bilateral (or quasi-bilinear) generating function. This quasi-bilateral generating function involving $p_n^{(\alpha)}(x)$, $q_m^{(n)}(u)$ is nothing but an extension of the bilateral generating function involving $p_n^{(\alpha)}(x)$. It may be pointed out that this extension is not unique because of the fact that when $q_m^{(n)}(u)$ is different from $p_m^{(n)}(u)$, the extension of the bilateral generating function is also different. In fact, when $q_m^{(n)}(u) = p_m^{(n)}(u)$, the extension of bilateral generating relation is quasi-bilinear. When $q_m^{(n)}(u)$ is different from $p_m^{(n)}(u)$, the extension is quasi-bilateral.

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