Journal of Inequalities and Special Functions
ISSN: 2217-4303, URL: http://www.ilirias.com
Volume 3 Issue 4, Pages 59-66.

A NEW SPECTRUM OF MOCK THETA FUNCTIONS OF ORDER TWO

PANKAJ SRIVASTAVA, ANWAR JAHAN WAHIDI

Abstract. The aim of the present paper is to established certain new representations of Mock theta functions of order two and relationships among the partial Mock theta functions and Mock theta functions of order two as well as relationships among the Hikamis Mock theta functions.

1. Introduction

The Mock theta functions, most famous and very interesting topic in the Ramanujans Lost note book, were first introduced by Ramanujan in his last letter to G.H. Hardy in January 1920. The celebrated letter contains a list of 17 functions that were classified into four of order three, ten of order five and three of order seven by Ramanujan though he gave no indication, what these orders were. G.N. Watson [21] in his famous lecture The final problem given on the occasion of his retirement as President of London Mathematical Society in 1935 introduced the mathematical world about a new class of functions that Ramanujan discovered and mentioned as Mock theta functions. After the discovery of Ramanujans Lost note book Mock theta functions were supplemented by Mock theta functions of order two, order six, order eight and order ten by Andrews and Hickerson [1], Gordon and McIntosh [11] and Choi [6] respectively. The beautiful aspects of these mock theta functions have been investigated extensively during the end of 20th century and even a number of remarkable results have been noticed in the beginning of 21st century. The transformation theory has played a vital role in enriching the literature of Mock theta functions. A number of representations of Mock theta functions given by R.P. Agarwal [2, 3, 4], Fine [10], R.Y. Denis and S.N. Singh [7], R.Y. Denis, S.N. Singh and S.P. Singh [8], R.Y. Denis, S.N. Singh and D. Sulata [9], Pankaj Srivastava [20], S. Ahmad Ali [5], A.K. Srivastava [16], Maheshwar Pathak and Pankaj Srivastava [15], Bhaskar Srivastava [17, 18] and etc. Recently McIntosh [14] has introduced a class of Mock theta functions and mentioned them Mock theta functions of order two. Later on Hikami [12, 13] in 2005 and 2006 gave new Mock theta functions of order four, order eight and order two respectively but he was silent about their relationship. Andrews attempted to develop double series representations of fifth, sixth and seventh order mock theta functions by making

---

2000 *Mathematics Subject Classification.* 33D15, 33D90.
Key words and phrases. Mock Theta Functions; Basic Hypergeometric Series.

Submitted 08. 11. 2012 . Published 10. 01. 2013 .

use of Heck operators. Later on B. Srivastava [17] used Bailey pair method to develop double series representations of one of the Mock theta function of order two and also developed relation between second order Mock theta functions $D_5(q)$ and $B(q)$ only.
In this paper, we have three aims; our first aim is to establish a bunch of double series representations of all Mock theta functions of order two in a very compact form by making use of identities due to H.M. Srivastava and V.K. Jain [19], Singh and Denis [7] respectively. Our second aim is to develop relationships among the partial Mock theta functions and Mock theta functions of order two making use of identity due to Srivastava [16]. As per the third aim, we have used the Srivastavas [16] identity again to develop relationship among the Mock theta functions $D_5(q)$,$D_6(q)$, $I_{12}(q)$ and $I_{13}(q)$ introduced by Hikami [12, 13].

2. Definitions and Notations

We shall use the following usual basic hypergeometric notations. The q-shifted factorial is defined by

for $|q| < 1$ and $|q^r| < 1$

$$(a;q)_n = \prod_{s=0}^{n-1}(1-aq^s),\ n \geq 1.$$

$$(a;q^r)_n = \prod_{s=0}^{n-1}(1-aq^{rs}),\ n \geq 1.$$

$$(a;q)_0 = 1,\ (a;q^r)_0 = 1.$$

$$(a;q^r)_\infty = \prod_{s=0}^{\infty}(1-aq^{rs}).$$

Definitions and Notations of Mock theta functions that shall be used in our analysis are as:

Mock theta functions of order two

Recently McIntosh[14] defined the second order mock theta functions:

$$A(q) = \sum_{n=0}^{\infty} \frac{q^{(n+1)^2}(-q;q^2)_n}{(q;q^2)^2_{(n+1)}} = \sum_{n=0}^{\infty} \frac{q^{(n+1)}(-q^2;q^2)_n}{(q;q^2)_{n+1}} \tag{2.1}$$

$$B(q) = \sum_{n=0}^{\infty} \frac{q^{n(n+1)}(-q^2;q^2)_n}{(q;q^2)^2_{n+1}} = \sum_{n=0}^{\infty} \frac{q^n(-q;q^2)_n}{(q;q^2)_{n+1}} \tag{2.2}$$

$$\mu(q) = \sum_{n=0}^{\infty} \frac{(-1)^n q^{n^2}(q;q^2)_n}{(-q^2;q^2)^2_n} \tag{2.3}$$

Hikami[13] introduced mock theta function of order two as:

$$D_5(q) = \sum_{n=0}^{\infty} \frac{q^n(-q;q)_n}{(q;q^2)_{n+1}} \tag{2.4}$$

Hikami [12] introduced mock theta functions of order four and order eight as:

$$D_6(q) = \sum_{n=0}^{\infty} \frac{q^n(-q^2;q^2)_n}{(q^{n+1};q)_{n+1}} \tag{2.5}$$

$$I_{12}(q) = \sum_{n=0}^{\infty} \frac{q^{2n}(-q;q^2)_n}{(q^{n+1};q)_{n+1}} \tag{2.6}$$

$$I_{13}(q) = \sum_{n=0}^{\infty} \frac{q^n(-q;q^2)_n}{(q^{n+1};q)_{n+1}} \tag{2.7}$$

$D_6(q)$ is of order four, $I_{12}(q)$ and $I_{13}(q)$ are of order eight.

If F(q)=$\sum_{n=0}^{\infty} f(q,n)$ is a mock theta function, then the corresponding partial mock theta function is denoted by the truncated series, $F_p(q) = \sum_{n=0}^{p} f(q,n)$.

3. We have used the following identities which are as follows:

The identity due to H.M. Srivastava and V.K. Jain [19] is as:

$$\sum_{l,m=0}^{\infty} \Omega_{l+m} \frac{(\lambda;q)_l(\mu;q)_m(\mu)^l z^{l+m}}{(q;q)_l(q;q)_m} = \sum_{n=0}^{\infty} \Omega_n \frac{(\lambda\mu;q)_n z^n}{(q;q)_n}, \tag{3.1}$$

where $(\Omega_n)_{n=0}^{\infty}$ is a bounded sequence of complex numbers and the parameter λ and μ are essentially arbitrary.
The identity due to S.N. Singh and Denis [7] is as:

$$\sum_{n=0}^{\infty} \frac{(aq/bc;q)_n \Omega_n x^n}{(q,aq/b,aq/c;q)_n} = \sum_{l,m=0}^{\infty} \frac{(a,bc;q)_m(1-aq^{2m})(-axq/bc)^m q^{m(m-1)/2}\Omega_{l+m}x^l}{(q,aq/b,aq/c;q)_m(1-a)(aq;q)_{2m+l}(q;q)_l} \tag{3.2}$$

The identity due to Srivastava [16] is as:

$$\sum_{r=0}^{n} \alpha_r \sum_{m=0}^{n} \delta_m = \sum_{m=0}^{n} \delta_m \sum_{r=0}^{m} \alpha_r + \sum_{r=0}^{n-1} \alpha_{r+1} \sum_{m=0}^{r} \delta_m. \tag{3.3}$$

4. **Main Results**

We have discussed the main results in three sections. The first section deals with alternative representations of Mock theta function of order two and in the second section, we have provided relations among the partial Mock theta functions and Mock theta functions of order two. Finally in the section three, we have discussed relationships among the newly introduced Hikamis mock theta function.

4.1. **New representations of Mock theta functions of order two.**

$$A(q) = \sum_{n=0}^{\infty} \frac{q^{(n+1)^2}(-q;q^2)_n}{(q;q^2)_{n+1}^2} = (1-q) \sum_{l,m=0}^{\infty} \frac{(-q;q^2)_{l+m} q^{(l+m+1)^2+l}}{(q;q^2)_{l+m+1}^2(1-q^{l+m+1})}. \tag{4.1}$$

$$A(q) = \sum_{n=0}^{\infty} \frac{q^{n+1}(-q^2;q^2)_n}{(q;q^2)_{n+1}} = (1-q) \sum_{l,m=0}^{\infty} \frac{(-q^2;q^2)_{l+m} q^{2l+m+1}}{(q;q^2)_{l+m+1}(1-q^{l+m+1})}. \tag{4.2}$$

$$A(q)=\sum_{n=0}^{\infty}\frac{(-q;q^2)_n q^{(n+1)^2}}{(q;q^2)_{n+1}^2}=\sum_{l,m=0}^{\infty}\frac{(-1)^m(-q;q^2)_{l+m}q^{(l+m+)^2+m(m+1)/2}}{(q;q^2)_{l+m+1}^2(q;q)_l(q;q)_m}. \quad (4.3)$$

$$A(q)=\sum_{n=0}^{\infty}\frac{(-q^2;q^2)_n q^{(n+1)}}{(q;q^2)_{n+1}}=\sum_{l,m=0}^{\infty}\frac{(-1)^m(-q^2;q^2)_{l+m}q^{(m^2+3m+2l+2)/2}}{(q;q^2)_{l+m+1}(q;q)_l(q;q)_m}. \quad (4.4)$$

$$B(q)=\sum_{n=0}^{\infty}\frac{q^{n(n+1)}(-q^2;q^2)_n}{(q;q^2)_{n+1}^2}=(1-q)\sum_{l,m=0}^{\infty}\frac{q^{(l+m)^2+2l+m}(-q^2;q^2)_{l+m}}{(1-q^{l+m+1})(q;q^2)_{l+m+1}^2}. \quad (4.5)$$

$$B(q)=\sum_{n=0}^{\infty}\frac{q^n(-q;q^2)_n}{(q;q^2)_{n+1}}=(1-q)\sum_{l,m=0}^{\infty}\frac{q^{2l+m}(-q;q^2)_{l+m}}{(1-q^{l+m+1})(q;q^2)_{l+m+1}}. \quad (4.6)$$

$$B(q)=\sum_{n=0}^{\infty}\frac{q^{n(n+1)}(-q^2;q^2)_n}{(q;q^2)_{n+1}^2}=\sum_{l,m=0}^{\infty}\frac{(-1)^m q^{(2l^2+3m^2+3m+2l+4lm)/2}}{(q;q^2)_{l+m+1}^2(q;q)_l(q;q)_m}. \quad (4.7)$$

$$B(q)=\sum_{n=0}^{\infty}\frac{q^n(-q;q^2)_n}{(q;q^2)_{n+1}}=\sum_{l,m=0}^{\infty}\frac{(-1)^m q^{(m^2+3m+2l)/2}}{(q;q^2)_{l+m+1}(q;q)_l(q;q)_m}. \quad (4.8)$$

$$\mu(q)=\sum_{n=0}^{\infty}\frac{(-1)^n q^{n^2}(q;q^2)_n}{(-q^2;q^2)_n^2}=(1-q)\sum_{l,m=0}^{\infty}\frac{(-1)^{l+m}q^{(l+m)^2+l}(q;q^2)_{l+m}}{(1-q^{l+m+1})(-q^2;q^2)_{l+m}^2}. \quad (4.9)$$

$$\begin{aligned}\mu(q)&=\sum_{n=0}^{\infty}\frac{(-1)^n q^{n^2}(q;q^2)_n}{(-q^2;q^2)_n^2}\\&=(1-q)\sum_{l,m=0}^{\infty}\frac{(-1)^{l+2m}q^{(2l^2+3m^2+4lm+m)/2}(q;q^2)_{l+m}}{(q;q)_l(q;q)_m(-q^2;q^2)_{l+m}^2}.\end{aligned} \quad (4.10)$$

$$D_5(q)=\sum_{n=0}^{\infty}\frac{q^n(-q;q)_n}{(q;q^2)_{n+1}}=\sum_{l,m=0}^{\infty}\frac{q^{2l+m}(q^2;q^2)_{l+m}}{(q;q^2)_{l+m+1}(q^2;q)_{l+m}}. \quad (4.11)$$

$$D_5(q)=\sum_{n=0}^{\infty}\frac{q^n(-q;q)_n}{(q;q^2)_{n+1}}=\sum_{l,m=0}^{\infty}\frac{(-1)^m q^{(m^2+3m+2l)/2}(q;q)_{l+m}}{(q;q^2)_{l+m+1}(q;q)_l(q;q)_m}. \quad (4.12)$$

4.2. **Relationship among the partial mock theta functions and mock theta functions of order two.**

$$D_{5n}(q)A_n(q)=\sum_{m=0}^{n}\frac{q^{(m+1)^2}(-q;q^2)_m}{(q;q^2)_{m+1}^2}D_{5m}(q)+\sum_{r=0}^{n-1}\frac{q^{(r+1)}(-q;q)_{r+1}}{(q;q^2)_{r+2}}A_r(q). \quad (4.13)$$

$$D_5(q)A(q)=\sum_{m=0}^{\infty}\frac{q^{(m+1)^2}(-q;q^2)_m}{(q;q^2)_{m+1}^2}D_{5m}(q)+\sum_{r=0}^{\infty}\frac{q^{(r+1)}(-q;q)_{r+1}}{(q;q^2)_{r+2}}A_r(q). \quad (4.14)$$

$$D_{5n}(q)B_n(q)=\sum_{m=0}^{n}\frac{q^{m(m+1)}(-q^2;q^2)_m}{(q;q^2)_{m+1}^2}D_{5m}(q)+\sum_{r=0}^{n-1}\frac{q^{(r+1)}(-q;q)_{r+1}}{(q;q^2)_{r+2}}B_r(q) \quad (4.15)$$

$$D_5(q)B(q) = \sum_{m=0}^{\infty} \frac{q^{m(m+1)}(-q^2;q^2)_m}{(q;q^2)^2_{m+1}} D_{5m}(q) + \sum_{r=0}^{\infty} \frac{q^{(r+1)}(-q;q)_{r+1}}{(q;q^2)_{r+2}} B_r(q). \tag{4.16}$$

$$D_{5n}(q)\mu_n(q) = \sum_{m=0}^{n} \frac{(-1)^m q^{m^2}(q;q^2)_m}{(-q^2;q^2)^2_m} D_{5m}(q) + \sum_{r=0}^{n-1} \frac{q^{(r+1)}(-q;q)_{r+1}}{(q;q^2)_{r+2}} \mu_r(q). \tag{4.17}$$

$$D_5(q)\mu(q) = \sum_{m=0}^{\infty} \frac{(-1)^m q^{m^2}(q;q^2)_m}{(-q^2;q^2)^2_m} D_{5m}(q) + \sum_{r=0}^{\infty} \frac{q^{(r+1)}(-q;q)_{r+1}}{(q;q^2)_{r+2}} \mu_r(q). \tag{4.18}$$

$$A_n(q)B_n(q) = \sum_{m=0}^{n} \frac{q^{m(m+1)}(-q^2;q^2)_m}{(q;q^2)^2_{m+1}} A_m(q) + \sum_{r=0}^{n-1} \frac{q^{(r+2)^2}(-q;q^2)_{r+1}}{(q;q^2)^2_{r+2}} B_r(q) \tag{4.19}$$

$$A(q)B(q) = \sum_{m=0}^{\infty} \frac{q^{m(m+1)}(-q^2;q^2)_m}{(q;q^2)^2_{m+1}} A_m(q) + \sum_{r=0}^{\infty} \frac{q^{(r+2)^2}(-q;q^2)_{r+1}}{(q;q^2)^2_{r+2}} B_r(q). \tag{4.20}$$

$$A_n(q)\mu_n(q) = \sum_{m=0}^{n} \frac{(-1)^m q^{m^2}(q;q^2)_m}{(-q^2;q^2)^2_m} A_m(q) + \sum_{r=0}^{n-1} \frac{q^{(r+2)^2}(-q;q^2)_{r+1}}{(q;q^2)^2_{r+2}} \mu_r(q). \tag{4.21}$$

$$A(q)\mu(q) = \sum_{m=0}^{\infty} \frac{(-1)^m q^{m^2}(q;q^2)_m}{(-q^2;q^2)^2_m} A_m(q) + \sum_{r=0}^{\infty} \frac{q^{(r+2)^2}(-q;q^2)_{r+1}}{(q;q^2)^2_{r+2}} \mu_r(q). \tag{4.22}$$

$$B_n(q)\mu_n(q) = \sum_{m=0}^{n} \frac{(-1)^m q^{m^2}(q;q^2)_m}{(-q^2;q^2)^2_m} B_m(q) + \sum_{r=0}^{n-1} \frac{q^{(r+1)(r+2)}(-q^2;q^2)_{r+1}}{(q;q^2)^2_{r+2}} \mu_r(q). \tag{4.23}$$

$$B(q)\mu(q) = \sum_{m=0}^{\infty} \frac{(-1)^m q^{m^2}(q;q^2)_m}{(-q^2;q^2)^2_m} B_m(q) + \sum_{r=0}^{\infty} \frac{q^{(r+1)(r+2)}(-q^2;q^2)_{r+1}}{(q;q^2)^2_{r+2}} \mu_r(q) \tag{4.24}$$

4.3. **Relationship among Hikami's mock theta functions.**

$$D_{5n}(q)D_{6n}(q) = \sum_{m=0}^{n} \frac{q^m(-q^2;q^2)_m}{(q^{m+1};q)_{m+1}} D_{5m}(q) + \sum_{r=0}^{n-1} \frac{q^{(r+1)}(-q;q)_{r+1}}{(q;q^2)_{r+2}} D_{6r}(q). \tag{4.25}$$

$$D_5(q)D_6(q) = \sum_{m=0}^{\infty} \frac{q^m(-q^2;q^2)_m}{(q^{m+1};q)_{m+1}} D_{5m}(q) + \sum_{r=0}^{\infty} \frac{q^{(r+1)}(-q;q)_{r+1}}{(q;q^2)_{r+2}} D_{6r}(q). \tag{4.26}$$

$$D_{5n}(q)I_{12n}(q) = \sum_{m=0}^{n} \frac{q^{2m}(-q;q^2)_m}{(q^{m+1};q)_{m+1}} D_{5m}(q) + \sum_{r=0}^{n-1} \frac{q^{(r+1)}(-q;q)_{r+1}}{(q;q^2)_{r+2}} I_{12r}(q). \tag{4.27}$$

$$D_5(q)I_{12}(q) = \sum_{m=0}^{\infty} \frac{q^{2m}(-q;q^2)_m}{(q^{m+1};q)_{m+1}} D_{5m}(q) + \sum_{r=0}^{\infty} \frac{q^{(r+1)}(-q;q)_{r+1}}{(q;q^2)_{r+2}} I_{12r}(q). \tag{4.28}$$

$$D_{5n}(q)I_{13n}(q) = \sum_{m=0}^{n} \frac{q^{m}(-q;q^2)_m}{(q^{m+1};q)_{m+1}} D_{5m}(q) + \sum_{r=0}^{n-1} \frac{q^{(r+1)}(-q;q)_{r+1}}{(q;q^2)_{r+2}} I_{13r}(q). \tag{4.29}$$

$$D_5(q)I_{13}(q) = \sum_{m=0}^{\infty} \frac{q^m(-q;q^2)_m}{(q^{m+1};q)_{m+1}} D_{5m}(q) + \sum_{r=0}^{\infty} \frac{q^{(r+1)}(-q;q)_{r+1}}{(q;q^2)_{r+2}} I_{13r}(q). \quad (4.30)$$

$$D_{6n}(q)I_{12n}(q) = \sum_{m=0}^{n} \frac{q^{2m}(-q;q^2)_m}{(q^{m+1};q)_{m+1}} D_{6m}(q) + \sum_{r=0}^{n-1} \frac{q^{(r+1)}(-q^2;q^2)_{r+1}}{(q^{r+2};q)_{r+2}} I_{12r}(q). \quad (4.31)$$

$$D_6(q)I_{12}(q) = \sum_{m=0}^{\infty} \frac{q^{2m}(-q;q^2)_m}{(q^{m+1};q)_{m+1}} D_{6m}(q) + \sum_{r=0}^{\infty} \frac{q^{(r+1)}(-q^2;q^2)_{r+1}}{(q^{r+2};q)_{r+2}} I_{12r}(q). \quad (4.32)$$

$$D_{6n}(q)I_{13n}(q) = \sum_{m=0}^{n} \frac{q^{m}(-q;q^2)_m}{(q^{m+1};q)_{m+1}} D_{6m}(q) + \sum_{r=0}^{n-1} \frac{q^{(r+1)}(-q^2;q^2)_{r+1}}{(q^{r+2};q)_{r+2}} I_{13r}(q). \quad (4.33)$$

$$D_6(q)I_{13}(q) = \sum_{m=0}^{\infty} \frac{q^{m}(-q;q^2)_m}{(q^{m+1};q)_{m+1}} D_{6m}(q) + \sum_{r=0}^{\infty} \frac{q^{(r+1)}(-q^2;q^2)_{r+1}}{(q^{r+2};q)_{r+2}} I_{13r}(q). \quad (4.34)$$

$$I_{12n}(q)I_{13n}(q) = \sum_{m=0}^{n} \frac{q^{m}(-q;q^2)_m}{(q^{m+1};q)_{m+1}} I_{12m}(q) + \sum_{r=0}^{n-1} \frac{q^{2(r+1)}(-q;q^2)_{r+1}}{(q^{r+2};q)_{r+2}} I_{13r}(q). \quad (4.35)$$

$$I_{12}(q)I_{13}(q) = \sum_{m=0}^{\infty} \frac{q^{m}(-q;q^2)_m}{(q^{m+1};q)_{m+1}} I_{12m}(q) + \sum_{r=0}^{\infty} \frac{q^{2(r+1)}(-q;q^2)_{r+1}}{(q^{r+2};q)_{r+2}} I_{13r}(q). \quad (4.36)$$

5. Proof of main results

5.1. **Proof of section (4.1).** As an illustration, we shall prove results (4.1) and (4.2).

To prove (4.1) replace Ω_n by $\Omega_n \frac{(a^2;q^2)_n}{(\lambda\mu;q)_n}$ in (3.1) and then we take $\lambda = \mu = q$ and $z = q$, after some simplification, identity (3.1) becomes

$$\sum_{n=0}^{\infty} \Omega_n (-q;q)_n q^n = \sum_{l,m=0}^{\infty} \Omega_{l+m} \frac{(q^2;q^2)_{l+m} q^{2l+m}}{(q^2;q)_{l+m}}. \quad (5.1)$$

Choosing $\Omega_n = \frac{(-q;q^2)_n q^{(n+1)^2-n}}{(q;q^2)^2_{n+1}(-q;q)_n}$ in (5.1) and simplifying further, we obtain the result (4.1).

To prove (4.2), assuming $a = bc$ and taking $b = c = 0$ in (3.2) and further putting x=q, we get

$$\sum_{n=0}^{\infty} \Omega_n q^n = \sum_{l,m=0}^{\infty} \Omega_{l+m} \frac{(-1)^m q^{l+m} q^{m(m+1)/2}}{(q;q)_l (q;q)_m}. \quad (5.2)$$

Taking $\Omega_n = \frac{(-q;q^2)_n q^{(n+1)^2-n}}{(q;q^2)^2_{n+1}}$ in (5.2) and after simplification we get the result (4.2).

Similarly, suitable selections of Ω_n and making use of respective identity one can establish the results (4.3) to (4.12).

5.2. **Proof of section (4.2 and 4.3).** As an illustration, we shall prove the result (4.13).

Taking $\alpha_r = \frac{q^r(-q;q)_r}{(q;q^2)_{r+1}}$ and $\delta_m = \frac{q^{(m+1)^2}(-q;q^2)_m}{(q;q^2)^2_{m+1}}$ in identity (3.3) and after simplification, we get (4.13),which provides the relationship among the partial Mock theta function of order two.

Again taking $lim_{n\to\infty}$ in (4.13), we obtain relation (4.14), which provides the relationship among partial Mock theta function and Mock theta function of order two. Choosing properly the value of α_r and δ_m and proceeding as above, one can obtain the results (4.15) to (4.36).

References

[1] G.E. Andrews and D. Hickerson,*Ramanujans LOST Note book II : The sixth order mock theta functions*, Adv. Math. 89 (1991), 60–105.

[2] R.P. Agarwal, *Ramanujans Mock Theta functions their Enegmatic Aspects* Proceeding of the International conference on the works of Srinivas Ramanujan (2000), 21-34.

[3] R.P. Agarwal, *An attempt towards presenting an unified theory for mock theta functions*, Proc.Int. Conf. SSFA1 (2001), 11–19.

[4] R.P. Agarwal, *Resonance of Ramanujans Mathematics II*, New Age International Pvt. Ltd. Publishers, New Delhi, 1996.

[5] S.Ahmad Ali, *A Bilateral expansion of Mock theta function of order two*, South East Asian J. Math. and Math. Sc. **vol 6**, No. 2 (2008).

[6] Y.S. Choi, *Tenth order mock theta functions in Ramanujan Lost Note Book*, Invent Math, **136** (1999), 497–596.

[7] R.Y. Denis and S.N. Singh, *Certain representation of mock theta functions of order six and ten*, Vikram Mathematical Journal 18 (1998), 1–15.

[8] Remy Denis, S.N. Singh and S.P. Singh,*On certain new representation of mock theta functions*, Italian J. Pure Appl. Math. 22 (2007), 149–154.

[9] R.Y. Denis, S.N. Singh and D. Sulata, *Certain Representation of mock theta functions*, Proc. International conference on Number Theory and Discrete Math. Chandigarh (India).

[10] N.J. Fine,*Basic Hypergeometric series and Applications, Mathematical Surveys and Monographs*, Amer. Math. Soc. 27 (1998).

[11] B. Gordon and R.J. McIntosh, *Some eighth order mock theta functions*, J. London Math. Soc. **(2)** 62 (2000), 321–335.

[12] Kazuhiro Hikami, *Mock (false) theta functions as quantum invariants*, Regular and Chaotic Dynamics, 10 (2005), 509–530.

[13] Kazuhiro Hikami, *Transformation formula of the 2nd order mock theta functions*, Lett. Math. Phys. 75 , 93–98 (2006).

[14] R. J. McIntosh, *Second order mock theta functions*, Canad. Math. Bull. 50 (2007), 284–290.

[15] Maheshwar Pathak and Pankaj Srivastava, *Certain relation for mock theta function of order eight*, Commun.Korean Math. Soc. **24** (2009), No.4, pp.629–640.

[16] A.K. Srivastava, *On partial sum of mock theta functions of order three*, Proc. Indian. Acad. Sci; **107** (1997), 1–12.

[17] B. Srivastava, *Partial second order mock theta functions their expansions and pade approximations*, J. Korean Math. Soc.**44** (2007), No.4, 767–777.

[18] B. Srivastava, *On a second order mock theta functions*, Tamsui Oxford Journal of Mathematical Sciences **24** (1) (2008) 25–35.

[19] H.M. Srivastava and V.K. Jain, *q-series identities and reducibility of basic double hypergeometric functions*, Can. J. Math. **37** (1) 1986, 215–231.

[20] Pankaj Srivastava, *Double series representation of mock theta functions*, South East Asian J. Math. and Math. Sc. **vol 1**, No. 3 (2003), 41–50.

[21] G.N. Watson, *The final problem, an account of the mock theta funtions*, J. London Math. Soc. 11 (1936),55–80.

PANKAJ SRIVASTAVA
Department of Mathematics,
Motilal Nehru National Institute of technology, Allahabad-211004, India.
E-mail address: drpankaj23@gmail.com

ANWAR JAHAN WAHIDI
Department of Mathematics,
Motilal Nehru National Institute of technology, Allahabad-211004, India.
E-mail address: anwar.wahidi@gmail.com