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GENERALIZED QUASI-CONTRACTION FOR DISLOCATED QUASI-METRIC SPACES

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Abstract. In this paper, T-orbitally complete dislocated quasi-metric spaces are utilized to validate fixed point results for freshly developed Geraghty quasi-contraction type mappings. The Geraghty quasi-contraction type mappings generalize Ciric's quasi-contraction mappings and other Geraghty quasi-contractive type mappings in the literature. Without establishing a continuity condition on the mapping, fixed point results are obtained, generalizing some relevant work in the literature.

1. Introduction

Geraghty[6] extended the Banach contraction mapping [7] in metric spaces by substituting an auxiliary function for a constant. Let Ω be the family of all functions $\beta : [0,\infty) \to [0,1)$ which satisfy the condition

$$\lim_{n\to\infty} \beta(t_n) = 1 \text{ implies } \lim_{n\to\infty} t_n = 0. \tag{1.1}$$

Using such a function, Geraghty [6] proved the following theorem.

Theorem 1.1. [6] *Let (X,d) be a complete metric space and T be a self-mapping on X. Suppose that there exists $\beta \in \Omega$ such that, for all $u,v \in X$,*

$$d(Tu,Tv) \leq \beta(d(u,v))d(u,v), \tag{1.2}$$

then T has a unique fixed point $z \in X$ and $\{T^n z\}$ converges to z for all $z \in X$.

Many extensions of Banach contraction mapping were investigated using several contractive assumptions (see [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20]).

Öztürk [21] introduced F-contraction and proved fixed point results for F-contractive iterates in a metric space. Some interesting results using other contractive conditions include [22, 23, 24, 25, 39, 40].

In the year 2000, Hitzler [27] proposed a space known as dislocated metric space, in which the self distance of points is not always zero, and established that the space's common Banach contraction mapping is also valid. Dislocated metric space

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is used in the semantic studies of logical programming, electronic engineering, and topology. Zeyada et al. [28] extended the concept of dislocated metric space by proposing dislocated quasi-metric space. The symmetric property is also omitted in this new definition. Other papers have been published that include fixed point results for self-mappings in metric spaces with various contraction conditions, as well as generalizations such as dislocated metric spaces and dislocated quasi-metric spaces (see [28, 29, 30, 31, 32, 33, 34, 35, 36, 37]).

Definition 1.2. [28] *Let X be a nonempty set and $d : X \times X \to \mathbb{R}^+$ be a function such that the following are satisfied:*

(i) $d(u, v) = d(v, u) = 0$ *implies that* $u = v$;
(ii) $d(u, v) \leq d(u, w) + d(w, v)$ *for all* $u, v, w \in X$.

Then d is called dislocated quasi-metric on X and the pair (X, d) is called a dislocated quasi-metric space.

Definition 1.3. [10] *Let $T : X \to X$ be a self-mapping and $\alpha : X \times X \to \mathbb{R}^+$ be a function. Then T is said to be α-orbital admissible if $\alpha(u, Tu) \geq 1$ implies $\alpha(Tu, T^2u) \geq 1$.*

Definition 1.4. [10] *Let $T : X \to X$ be a self-mapping and $\alpha : X \times X \to \mathbb{R}^+$ be a function. Then T is said to be triangular α-orbital admissible if T is α-orbital admissible and $\alpha(u, v) \geq 1$, $\alpha(v, Tv) \geq 1$ imply $\alpha(u, Tv) \geq 1$.*

Lemma 1.5. [10] *Let $T : X \to X$ be a triangular α-orbital admissible mapping. Assume that there exists $u_1 \in X$ such that $\alpha(u_1, Tu_1) \geq 1$. Define a sequence $\{u_n\}$ by $u_{n+1} = Tu_n$. Then, we have $\alpha(u_n, u_m) \geq 1$ for all $m, n \in \mathbb{N}$ with $n < m$.*

Definition 1.6. [36] *Let $T : X \to X$ be a selfmapping on a metric space. For each $u \in X$ and for any positive whole number n,*

$$O_T(u, n) = \{u, Tu, \ldots, T^n u\} \quad \textit{and} \quad O_T(u, \infty) = \{u, Tu, \ldots, T^n u, \ldots\}.$$

The set $O_T(u, \infty)$ is called the orbit of T at u and the metric space X is called T-orbitally complete if every Cauchy sequence in $O_T(u, \infty)$ is convergent in X.

It is clear that every complete dislocated quasi-metric space is T-orbitally complete. But the converse does not hold in general.

The purpose of this paper is to prove some fixed point results in dislocated quasi-metric space using Geraghty type generalized quasi-contraction. The result is obtained by removing the continuity constraint and proving the presence and uniqueness of a fixed point in a dislocated quasi-metric space that is orbitally complete (which is a relaxation of completeness). This finding generalizes many previous studies in the field [6, 7, 8, 9, 10, 27, 28, 33, 34, 35, 36, 37, 38, 39].

2. Main results

Let Ψ denote the class of the functions $\psi : [0, \infty) \to [0, \infty)$ which satisfy the following conditions:

(a) ψ is non-decreasing;
(b) ψ is continuous;
(c) $\psi(t) = 0 \Leftrightarrow t = 0$.

Definition 2.1. *Let (X,d) be a dislocated quasi-metric space, and $\alpha : X \times X \to \mathbb{R}^+$ be a function. A self-mapping $T : X \to X$ is called an (α,ψ,β)-Geraghty type contraction mapping if there exists $\beta \in \Omega$ such that, for all $u,v \in X$ with $d(Tu,Tv) > 0$ and $\alpha(u,v) \geq 1$,*

$$\alpha(u,v)\psi(d(Tu,Tv)) \leq \beta(\psi(M_T(u,v)))\psi(M_T(u,v)), \tag{2.1}$$

where

$$M_T(u,v) = \max\left\{d(u,v), d(u,Tu), d(v,Tv), \frac{(1+d(u,Tv))d(v,Tu)}{1+d(u,v)}, \frac{(1+d(u,Tu))d(v,Tv)}{1+d(u,v)}\right\}.$$

Theorem 2.2. *Let (X,d) be a T-orbitally complete dislocated quasi-metric space such that $T : X \to X$ is a self-mapping. Suppose $\alpha : X \times X \to \mathbb{R}^+$ is a function satisfying the following conditions:*

(i) *T is an (α,ψ,β)-Geraghty type contraction mapping*;
(ii) *T is triangular α-orbital admissible mapping*;
(iii) *There exists $u_1 \in X$ such that $\alpha(u_1,Tu_1) \geq 1$.*

Then T has a fixed point $z \in X$ and $\{T^n u_1\}$ converges to z.

Proof. Let $u_1 \in X$ such that $\alpha(u_1,Tu_1) \geq 1$. Define a sequence $\{u_n\}$ by $u_{n+1} = T^n u$, for $n \geq 1$. If $u_n = u_{n+1}$ for some n, then obviously T has a fixed point. Consequently, throughout the proof, we suppose that $u_n \neq u_{n+1}$ for all $n \geq 1$. By Lemma 1.5, used recursively, we have

$$\alpha(u_n,u_{n+1}) \geq 1 \text{ for all } n \geq 1. \tag{2.2}$$

By (2.1), we get

$$\begin{aligned}\psi(d(u_{n+1},u_{n+2})) &= \psi(d(T^n u, T^{n+1}u)) \\ &\leq \alpha(T^{n-1}u,T^n u)\psi(d(TT^{n-1}u,TT^n u)) \\ &\leq \beta(\psi(M_T(T^{n-1}u,T^n u)))\psi(M_T(T^{n-1}u,T^n u)),\end{aligned} \tag{2.3}$$

where

$$\begin{aligned}&M_T(T^{n-1}u,T^n u) \\ &= \max\Bigg\{d(T^{n-1}u,T^n u), d(T^{n-1}u,T^n u), d(T^n u,T^{n+1}u), \\ &\quad \frac{(1+d(T^{n-1}u,T^{n+1}u)d(T^n u,T^n u)}{1+d(T^{n-1}u,T^n u)}, \frac{(1+d(T^{n-1}u,T^n u))d(T^n u,T^{n+1}u)}{1+d(T^{n-1}u,T^n u)}\Bigg\} \\ &= \max\left\{d(T^{n-1}u,T^n u), d(T^n u,T^{n+1}u)\right\}.\end{aligned}$$

Assume that $\psi(M_T(T^{n-1}u,T^n u)) = \psi(d(T^n u,T^{n+1}u))$ and from (2.3), we get

$$\begin{aligned}\psi(d(T^n u,T^{n+1}u)) &\leq \alpha(T^{n-1}u,T^n u)\psi(d(TT^{n-1}u,TT^n u)) \\ &\leq \beta(\psi(M_T(T^{n-1}u,T^n u)))\psi(M_T(T^{n-1}u,T^n u)) \\ &= \beta(\psi(d(T^n u,T^{n+1}u)))\psi(d(T^n u,T^{n+1}u)) \\ &< \psi(d(T^n u,T^{n+1}u)),\end{aligned} \tag{2.4}$$

which is a contradiction. Thus, $\psi(M_T(T^{n-1}u, T^n u)) \neq \psi(d(T^n u, T^{n+1}u))$. Next, assume that $\psi(M_T(T^{n-1}u, T^n u)) = \psi(d(T^{n-1}u, T^n u))$ and from (2.3), we get

$$\begin{aligned}\psi(d(T^n u, T^{n+1}u)) &\leq \alpha(T^{n-1}u, T^n u)\psi(d(TT^{n-1}u, TT^n u)) \\ &\leq \beta(\psi(M_T(T^{n-1}u, T^n u)))\psi(M_T(T^{n-1}u, T^n u)) \\ &= \beta(\psi(d(T^{n-1}u, T^n u)))\psi(d(T^{n-1}u, T^n u)) \\ &< \psi(d(T^{n-1}u, T^n u)).\end{aligned} \tag{2.5}$$

Then, $\psi(d(T^n u, T^{n+1}u)) < \psi(d(T^{n-1}u, T^n u))$. Thus, the sequence $\{d(T^n u, T^{n+1}u)\}$ is positive and decreasing. Consequently, there exists $k \geq 0$ such that

$$\lim_{n\to\infty} d(T^n u, T^{n+1}u) = k.$$

We claim that $k = 0$. Suppose, on the contrary, that $k > 0$. Then, from (2.3) we have

$$\frac{\psi(d(T^n u, T^{n+1}u))}{\psi(M_T(T^{n-1}u, T^n u))} \leq \beta(\psi(M_T(T^{n-1}u, T^n u))) < 1 \tag{2.6}$$

and

$$\lim_{n\to\infty} \beta(\psi(M_T(T^{n-1}u, T^n u))) = 1. \tag{2.7}$$

Because, $\beta \in \Omega$, by definition, it implies that

$$\lim_{n\to\infty} \psi(M_T(T^{n-1}u, T^n u)) = 0. \tag{2.8}$$

and so

$$\lim_{n\to\infty} d(T^n u, T^{n+1}u) = 0, \tag{2.9}$$

which is a contradiction.

Suppose that the sequence $\{u_n\}$ is not a Cauchy, then there exists $\epsilon > 0$ and we can define two subsequences $\{T^{m_l}u\}$ and $\{T^{n_l}u\}$ of the sequence $\{T^n u\}$ such that, for any $n_l > m_l > l$, $d(T^{m_l}u, T^{n_l}u) \geq \epsilon$, but $d(T^{m_l}u, T^{n_l-1}u) < \epsilon$, we observe that

$$\begin{aligned}\epsilon &\leq d(T^{m_l}u, T^{n_l}u) \\ &\leq d(T^{m_l}u, T^{n_l-1}u) + d(T^{n_l-1}u, T^{n_l}u) \\ &\leq d(T^{m_l}u, T^{m_l-1}u) + d(T^{m_l-1}u, T^{n_l-1}u) + d(T^{n_l-1}u, T^{n_l}u) \\ &\leq d(T^{m_l}u, T^{m_l-1}u) + d(T^{m_l-1}u, T^{n_l}u) + 2d(T^{n_l-1}u, T^{n_l}u) \\ &< d(T^{m_l}u, T^{m_l-1}u) + \epsilon + 2d(T^{n_l-1}u, T^{n_l}u).\end{aligned} \tag{2.10}$$

Since $d(T^n u, T^{n+1}u) \neq 0$, we get

$$\begin{aligned}\lim_{l\to\infty} d(T^{m_l}u, T^{n_l}u) &= \lim_{l\to\infty} d(T^{m_l}u, T^{n_l-1}u) \\ &= \lim_{l\to\infty} d(T^{m_l-1}u, T^{n_l-1}u) \\ &= \lim_{l\to\infty} d(T^{m_l-1}u, T^{n_l}u) \\ &= \epsilon.\end{aligned} \tag{2.11}$$

From T is an (α, ψ, β)-Geraghty type contraction mapping and $\alpha(u, v) \geq 1$, we obtain

$$\begin{aligned}\psi(d(T^{m_l}u, T^{n_l}u)) &\leq \alpha(T^{m_l-1}u, T^{n_l-1}u)\psi(d(TT^{m_l-1}u, TT^{n_l-1}u)) \\ &\leq \beta(\psi(M_T(T^{m_l-1}u, T^{n_l-1}u)))\psi(M_T(T^{m_l-1}u, T^{n_l-1}u)),\end{aligned} \tag{2.12}$$

where

$$
\begin{aligned}
&M_T(T^{m_l-1}u, T^{n_l-1}u)\\
&= \max\Big\{ d(T^{m_l-1}u, T^{n_l-1}u), d(T^{m_l-1}u, T^{m_l}u), d(T^{n_l-1}u, T^{n_l}u),\\
&\frac{(1+d(T^{m_l-1}u, T^{n_l}u))d(T^{n_l-1}u, T^{m_l}u)}{1+d(T^{m_l-1}u, T^{n_l-1}u)}, \frac{(1+d(T^{m_l-1}u, T^{m_l}u))d(T^{n_l-1}u, T^{n_l}u)}{1+d(T^{m_l-1}u, T^{n_l-1}u)}\Big\}.
\end{aligned} \tag{2.13}
$$

Letting $l \to \infty$ in (2.13) and using (2.11), we obtain

$$\lim_{l\to\infty} M_T(T^{m_l-1}u, T^{n_l-1}u) = \epsilon. \tag{2.14}$$

From (2.12), we get

$$\psi(\epsilon) \leq \lim_{l\to\infty} \beta(\psi(M_T(T^{m_l-1}u, T^{n_l-1}u)))\psi(\epsilon) \tag{2.15}$$

and

$$1 \leq \lim_{l\to\infty} \beta(\psi(M_T(T^{m_l-1}u, T^{n_l-1}u))). \tag{2.16}$$

Thus, $\lim_{l\to\infty} \beta(\psi(M_T(T^{m_l-1}u, T^{n_l-1}u))) = 1$ and hence

$$\lim_{l\to\infty} \psi(M_T(T^{m_l-1}u, T^{n_l-1}u)) = 0.$$

Therefore

$$\lim_{l\to\infty} d(T^{m_l-1}u, T^{n_l-1}u) = 0, \tag{2.17}$$

which is a contradiction. So, it follows that $\{T^n u\}$ is a Cauchy sequence. From T-orbitally complete, there exists $z \in X$ such that $T^n u \to z$ as $n \to \infty$. To show that $Tz = z$, suppose that

$$d(z, Tz) = \lim_{n\to\infty} d(T^n u, Tz) > 0.$$

We have

$$
\begin{aligned}
\psi(d(u_{n+1}, Tz)) &= \psi(d(T^n u, Tz))\\
&\leq \alpha(T^{n-1}u, z)\psi(d(T^n u, Tz))\\
&\leq \beta(\psi(M_T(T^{n-1}u, z)))\psi(M_T(T^{n-1}u, z)),
\end{aligned} \tag{2.18}
$$

where

$$
\begin{aligned}
&M_T(T^{n-1}u, z)\\
&= \max\Big\{ d(T^{n-1}u, z), d(T^{n-1}u, T^n u), d(z, Tz), \frac{(1+d(T^{n-1}u, Tz))d(z, T^n u)}{1+d(T^{n-1}u, z)},\\
&\frac{(1+d(T^{n-1}u, T^n u))d(z, Tz)}{1+d(T^{n-1}u, z)}\Big\}.
\end{aligned}
$$

Letting $n \to \infty$, we get

$$\begin{aligned}
&\lim_{n\to\infty} M_T(T^{n-1}u, z)\\
&= \max\left\{d(z,z), d(z,z), d(z,Tz), \frac{(1+d(z,Tz))d(z,z)}{1+d(z,z)}, \frac{(1+d(z,z))d(z,Tz)}{1+d(z,z)}\right\}\\
&= \max\left\{d(z,Tz), 0\right\}\\
&= d(z,Tz).
\end{aligned}$$

Hence, by letting the limits as $n \to \infty$ in (2.18), we get

$$\begin{aligned}
\psi(d(z,Tz)) &\leq \beta(\psi(d(z,Tz)))\psi(d(z,Tz))\\
&< \psi(d(z,Tz)),
\end{aligned}$$

which is a contradiction. Therefore, we obtain $d(z,Tz) = 0$. Similarly, $d(Tz,z) = 0$. That is, $z = Tz$ and the fixed point of T is z. □

Theorem 2.3. *All the conditions of Theorem 2.2, we find that z is a unique fixed point of T.*

Proof. From the proof of Theorem 2.2, z is a fixed point of T. Assume that z and w are distinct fixed points of T. By condition (ii) in Theorem 2.2, we get

$$\begin{aligned}
\psi(d(z,w)) &= \psi(d(Tz,Tw))\\
&\leq \alpha(z,w)\psi(d(Tz,Tw))\\
&\leq \beta(\psi(M_T(z,w)))\psi(M_T(z,w)),
\end{aligned}$$

where

$$\begin{aligned}
&M_T(z,w)\\
&= \max\left\{d(z,w), d(z,Tz), d(w,Tw), \frac{(1+d(z,Tw))d(w,Tz)}{1+d(z,w)}, \frac{(1+d(z,Tz))d(w,Tw)}{1+d(z,w)}\right\}\\
&= d(z,w).
\end{aligned}$$

Thus,

$$\begin{aligned}
\psi(d(z,w)) &\leq \beta(\psi(d(z,w)))\psi(d(z,w))\\
&< \psi(d(z,w)),
\end{aligned}$$

which is a contradiction. So, $z = w$. Hence, T has a unique fixed point. □

Corollary 2.4. *Let (X,d) be a T-orbitally complete dislocated quasi-metric space and $\alpha : X \times X \to \mathbb{R}^+$ is a function. Suppose there exist $\beta \in \Omega$ and $\psi \in \Psi$ such that, for all $u, v \in X$ with $d(Tu,Tv) > 0$ and $\alpha(u,v) \geq 1$,*

$$\alpha(u,v)\psi(d(Tu,Tv)) \leq \beta(\psi(M_T(u,v)))\psi(M_T(u,v)),$$

where

$$M_T(u,v) = \max\{d(u,v), d(u,Tu), d(v,Tv)\},$$

and satisfying the following conditions:

(i) *T is triangular α-orbital admissible mapping.*
(ii) *There exists $u_1 \in X$ such that $\alpha(u_1, Tu_1) \geq 1$.*

Then T has a fixed point $z \in X$ and $\{T^n u_1\}$ converges to z.

Proof. We obtain the proof by following the proof in Theorem 2.2. □

Corollary 2.5. *Let (X, d) be a T-orbitally complete dislocated quasi-metric space and $\alpha : X \times X \to \mathbb{R}^+$ is a function. Suppose there exist $\beta \in \Omega$ and $\psi \in \Psi$ such that, for all $u, v \in X$ with $d(Tu, Tv) > 0$ and $\alpha(u, v) \geq 1$,*

$$\alpha(u, v)\psi(d(Tu, Tv)) \leq \beta(\psi(d(u, v)))\psi(d(u, v)),$$

and satisfying the following conditions:

(i) *T is triangular α-orbital admissible mapping.*
(ii) *There exists $u_1 \in X$ such that $\alpha(u_1, Tu_1) \geq 1$.*

Then T has a fixed point $z \in X$ and $\{T^n u_1\}$ converges to z.

Proof. We obtain the proof by following the proof in Theorem 2.2. □

Corollary 2.6. *Let (X, d) be a T-orbitally complete dislocated quasi-metric space. Suppose there exist $\beta \in \Omega$ and $\psi \in \Psi$ such that, for all $u, v \in X$ with $d(Tu, Tv) > 0$,*

$$\psi(d(Tu, Tv)) \leq \beta(\psi(M_T(u, v)))\psi(M_T(u, v)),$$

where

$$M_T(u, v) = \max\left\{d(u, v), d(u, Tu), d(v, Tv), \frac{(1 + d(u, Tv))d(v, Tu)}{1 + d(u, v)}, \frac{(1 + d(u, Tu))d(v, Tv)}{1 + d(u, v)}\right\}.$$

Then T has a fixed point $z \in X$ and $\{T^n u_1\}$ converges to z.

Proof. Letting $\alpha(u, v) = 1$ and we obtain the proof by following the proof in Theorem 2.2. □

Corollary 2.7. *Let (X, d) be a T-orbitally complete dislocated quasi-metric space. Suppose there exist $\beta \in \Omega$ and $\psi \in \Psi$ such that, for all $u, v \in X$ with $d(Tu, Tv) > 0$,*

$$\psi(d(Tu, Tv)) \leq \beta(\psi(M_T(u, v)))\psi(M_T(u, v)),$$

where

$$M_T(u, v) = \max\{d(u, v), d(u, Tu), d(v, Tv)\}.$$

Then T has a fixed point $z \in X$ and $\{T^n u_1\}$ converges to z.

Proof. Letting $\alpha(u, v) = 1$ and we obtain the proof by following the proof in Theorem 2.2. □

Corollary 2.8. *Let (X, d) be a T-orbitally complete dislocated quasi-metric space. Suppose there exist $\beta \in \Omega$ and $\psi \in \Psi$ such that, for all $u, v \in X$ with $d(Tu, Tv) > 0$,*

$$\psi(d(Tu, Tv)) \leq \beta(\psi(d(u, v)))\psi(d(u, v)).$$

Then T has a fixed point $z \in X$ and $\{T^n u_1\}$ converges to z.

Proof. Letting $\alpha(u, v) = 1$ and we obtain the proof by following the proof in Theorem 2.2. □

Example 2.9. *Let $X = [0, \infty)$ and $d(u, v) = u$ for all $u, v \in X$. Let $\beta(t) = \frac{1}{1+t}$ for all $t > 0$. Then $\beta \in \Omega$. Let $\psi(t) = 3t$ and a mapping $T : X \to X$ be defined by*

$$T(u) = \begin{cases} \frac{1}{4}u, & \text{if } u \in [0, 1] \\ 3u, & \text{if } u > 1. \end{cases}$$

Define a function $\alpha : X \times X \to [0, \infty)$ *by*

$$\alpha(u,v) = \begin{cases} 1, & \text{if } u,v \in [0,1] \\ 0, & \text{otherwise.} \end{cases}$$

Condition (iii) of Theorem 2.2 is satisfied with $u_1 = 1$.

For condition (ii), let u, v be such that $\alpha(u,v) \geq 1$. Then, $u, v \in [0,1]$ and $Tu, Tv \in [0,1]$. Moreover, $\alpha(v, Tv) = \alpha(u, Tu) = 1$ and $\alpha(Tu, T^2u) = 1$. So, T is triangular α-orbital admissible. Thus, (ii) is satisfied.

Finally, we show that condition (i) is satisfied.

Case (i): If $u, v \in [0,1]$, then $\alpha(u,v) = 1$ and

$$\begin{aligned} &\beta(\psi(M_T(u,v)))\psi(M_T(u,v)) - \alpha(u,v)\psi(d(Tu,Tv)) \\ &= \beta(\psi(M_T(u,v)))\psi(M_T(u,v)) - \psi(d(Tu,Tv)) \\ &= \frac{3M_T(u,v)}{1+3M_T(u,v)} - 3Tu \\ &\geq 0. \end{aligned}$$

Thus, $\alpha(u,v)\psi(d(Tu,Tv)) \leq \beta(\psi(M_T(u,v)))\psi(M_T(u,v))$ for $u, v \in [0,1]$.

Case (ii): If $u \in [0,1], v > 1$ or $u, v > 1$ then, obviously, $\alpha(u,v) = 0$ and we have

Thus, all assumptions of Theorem 2.2 and Theorem 2.3 are satisfied, and hence T has a unique fixed point $z = 0$.

3. Conclusions

We study fixed point theory on X be a T-orbitally complete dislocated quasi-metric space by generalized quasi-contraction, Geraghty contraction, and admissible function.

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