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IDEAL CONVERGENCE VIA REGULAR MATRIX SUMMABILTY METHOD

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Abstract. In this paper, we apply the notion of $\mathfrak{B}$-summability to define a more general case of ideal convergence. We study several properties of this new summability method.

1. Introduction and Preliminaries

The $\Im$-convergence was introduced in [15] which is more general than statistical convergence (see [10] and [33]). Recently Balcerzak et al [1] and Komisarski [18] have studied $\Im$-convergence for functions. Further this notion was studied by several authors, e.g. [4], [5], [24], [27]-[30]. Our aim of this paper is to define the most general case of $\Im$-convergence through a sequence of regular matrices.

Let $G \subseteq \mathbb{N}$ (the set of +ve integers). The natural density (or asymptotic density or arithmetic density) *of* G is defined by

$$\delta(G) = \lim_{n\to\infty} \frac{\#\{r \leq n : r \in G\}}{n} = 0$$

where # indicates the cardinality of the enclosed set.

Let $G_{(\varepsilon)} = \{r \leq n : |\xi_r - l| \geq \varepsilon\}$. If $\delta(G_{(\varepsilon)}) = 0$ for each $\varepsilon > 0$, then the sequence $\xi = (\xi_r)$ is said to be statistically convergent to the number l, i.e. $S\text{-}\lim \xi = l$.

Let $A = (f_{nk})_{n,k=1}^{\infty}$ be a non-negative regular matrix. Then the A-density [11] of the set $G \subseteq \mathbb{N}$ is defined by

$$\delta_A(G) := \lim_n \sum_{k\in G} f_{nk} \text{ exists.}$$

If $\delta_A(G_{(\varepsilon)}) = 0$ for each $\varepsilon > 0$ then $\xi = (\xi_r)$ is said to be A-statistically convergent to the number l (cf. [16]) which is written as $S_A\text{-}\lim \xi = l$.

Kolk [17] defined $\mathfrak{B}$-statistical convergence, a more general notion, via the concept of $F_{\mathfrak{B}}$-convergence (see [32]).

Let $\mathfrak{B} = (B_j) = ([b_{nk}(j)])$. A sequence $\xi = (\xi_r) \in \ell_\infty$ is called $F_{\mathfrak{B}}$-convergent to the value $\mathfrak{L}$ if $\lim_n (B_j\xi)_n = \lim_n \sum_k b_{nk}(j)\xi_k = \mathfrak{L}$, uniformly in $j \in \mathbb{N}$. In this case $\mathfrak{L}$ is called the $\mathfrak{B}$-$\lim \xi$.

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If $\delta_{\mathfrak{B}}(G) = \mathfrak{p}$ then we say that an index set G has $\mathfrak{B}$-density equal to $\mathfrak{p}$ (see Kolk [17]), i.e.

$$\lim_n \sum_{k\in G} b_{nk}(j) = \mathfrak{p} \text{ uniformly in } j,$$

where $G = \{k_l\} \subset \mathbb{N}$, $k_l < k_{l+1}$ $\forall$ l.

Let $\mathfrak{B} \in \mathcal{R}_+$, the set of non-negative regular methods $\mathfrak{B}$. A sequence $\xi = (\xi_r)$ is called $\mathfrak{B}$-statistically convergent to the number $\mathfrak{L}$, if $\delta_{\mathfrak{B}}(G_{(\varepsilon)}) = 0$ for every $\varepsilon > 0$ and we denote it by $S_{\mathfrak{B}}\text{-}\lim \xi = \mathfrak{L}$. See also [25] and [7].

For $\mathfrak{B} = (C,1)$, we get statistical convergence. In case $\mathfrak{B} = A$, we get A-statistical convergence.

2. Ideal convergence and its generalization

If $\mathcal{Y} \neq \varnothing$, then a family $\Im \subset 2^{\mathcal{Y}}$ is an ideal $\Leftrightarrow$ (i) $\varnothing \in \Im$, (ii) $E, F \in \Im \Rightarrow E \cup F \in \Im$, and (iii) $E \in \Im$ and $F \subset E \Rightarrow F \in \Im$.

An ideal $\Im$ is called nontrivial ideal if $\Im \neq \varnothing$ and $\mathcal{Y} \notin \Im$.

$\mathcal{F}(\neq \varnothing) \subset 2^{\mathcal{Y}}$ is called the filter on $\mathcal{Y} \Leftrightarrow$ (i) $\varnothing \notin \mathcal{F}$, (ii) $E, F \in \mathcal{F} \Rightarrow E \cap F \in \mathcal{F}$, and (iii) $E \in \mathcal{F}$ and $F \supset E \Rightarrow F \in \mathcal{F}$

If a non-trivial ideal $\Im$ in $\mathcal{Y}$ contains all singletons then it is called an admissible, and vice-versa.

For a non-trivial ideal $\Im \subset 2^{\mathcal{Y}}$, a class $\mathcal{F}(\Im) = \{M \subset \mathcal{Y} : M = \mathcal{Y} \backslash \mathcal{K}, \text{ for some } \mathcal{K} \in \Im\}$ is called the filter associated with the ideal $\Im$.

A sequence $\xi = (\xi_k)$ is $\Im$-convergent to l if for every $\varepsilon > 0$,

$$\{k \in \mathbb{N} : |\xi_k - l| \geq \varepsilon\} \in \Im,$$

which is written as $\Im\text{-}\lim \xi = l$.

In [14] defined $\mathfrak{B}(\Im)$-statistically convergent and studied some properties.

Definition 2.1. *A sequence $\xi = (\xi_k)$ is called $\mathfrak{B}(\Im)$-statistically convergent to the number $\mathfrak{L}$, if for every $\varepsilon > 0$ and $d > 0$*

$$\{n \in \mathbb{N} : \sum_{k\in G_{(\varepsilon)}} b_{nk}(j) \geq d\} \in \Im$$

uniformly in j, where $G_{(\varepsilon)} = \{k \in \mathbb{N} : |\xi_k - \mathfrak{L}| \geq \varepsilon\}$ and $\mathfrak{B} \in \mathcal{R}_+$. We write $S_{\mathfrak{B}}(\Im)\text{-}\lim \xi = \mathfrak{L}$. Such a class of sequences will be denoted by $S_{\mathfrak{B}}(\Im)$.

One can easily see that for different choice of the matrix sequence $\mathfrak{B}$ and $\Im$ we get different types of statistical methods, e.g. statistical convergence [12], lacunary statistical convergence [13], λ-statistical convergence [22], λ-statistical convergence of order α [2], $\Im$-convergence [15], $\Im$-λ-statistical convergence [30], $\Im$-lacunary statistical convergence [3], $A(\Im)$ statistical convergence [31].

Now we define $\mathfrak{B}(\Im)$-summable, $\mathfrak{B}(\Im^*)$-summable and statistically $\mathfrak{B}(\Im)$-summable.

Definition 2.2. *Let $\mathfrak{B} \in \mathcal{R}_+$. A sequence $\xi = (\xi_k)$ is said to be $\mathfrak{B}(\Im)$-summable to the number $\mathfrak{L}$, i.e. $\mathfrak{B}(\Im)\text{-}\lim \xi = \mathfrak{L}$, if $\forall$ $\varepsilon > 0$,*

$$\{n \in \mathbb{N} : |(B_j\xi)_n - \mathfrak{L}| \geq \varepsilon\} \in \Im$$

uniformly in j. Such a class of sequences will be denoted by $\mathfrak{B}(\Im)$.

Definition 2.3. *Let $\mathfrak{B} \in \mathcal{R}_+$. A sequence $\xi = (\xi_k)$ is called $\mathfrak{B}(\Im^*)$-summable to the number $\mathfrak{L}$, if there exists $\mathfrak{K} = \{k_m : k_1 < k_2 < \cdots\} \subset \mathbb{N}$ such that $\mathfrak{K} \in \mathcal{F}(\Im)$ and $\mathfrak{B}\text{-}\lim_m \xi_{k_m} = \lim_m (B_j\xi)_m = \mathfrak{L}$, uniformly in j. In this case we write $\mathfrak{B}(\Im^*)\text{-}\lim \xi = \mathfrak{L}$.*

For $\mathfrak{B} = A$, $\mathfrak{B}(\Im)$ and $\mathfrak{B}(\Im^*)$-summable becomes $A(\Im)$ and $A(\Im^*)$ summable due to [31] and [8].

Definition 2.4. *Let $\mathfrak{B} \in \mathcal{R}_+$. A sequence $\xi = (\xi_k)$ is said to be statistically $\mathfrak{B}(\Im)$-summable to the number $\mathfrak{L}$, i.e. $\Im(S_{\mathfrak{B}})\text{-}\lim \xi = \mathfrak{L}$, if $\forall\, \varepsilon > 0$ and $d > 0$,*

$$\{n \in \mathbb{N} : \frac{\#\{t \leq n : |(B_j\xi)_t - \mathfrak{L}| \geq \varepsilon\}}{n} \geq d\} \in \Im,$$

uniformly in j. Such a class of sequences will be denoted by $\Im(S_{\mathfrak{B}})$.

For $\mathfrak{B} = A$, statistical $\mathfrak{B}(\Im)$-summability is reduced to statistical $A(\Im)$-summability [9] and if $\Im = \Im_{fin} = \{G \subseteq \mathbb{N} : G \text{ is finite}\}$, statistical $\mathfrak{B}(\Im)$-summability is reduced to statistical A-summability [6]. Further for different choice of A, we get statistical summability $(C,1)$ due to Moricz [19], statistical summability $(H,1)$ due to Moricz [20], statistical summability $(\bar{N},p)$ (see [21]), statistical σ-convergence [26] and statistical λ-summability [23].

3. Main results

We establish some relations here.

Theorem 3.1. *For an admissible ideal $\Im$, the $\mathfrak{B}(\Im)$-statistical convergence of $\xi = (\xi_k) \in \ell_\infty \Rightarrow \mathfrak{B}(\Im)$-summability but the converse need not hold.*

Proof. Let $\xi = (\xi_k) \in \ell_\infty$ be $\mathfrak{B}(\Im)$-statistically convergent to $\mathfrak{L}$. Let for $\varepsilon > 0$, $K_{(\varepsilon)} = \{k \in \mathbb{N} : |\xi_k - \mathfrak{L}| \geq \varepsilon\}$. Then

$$\begin{aligned} |(B_j\xi)_n - \mathfrak{L}| &\leq |\sum_{k \notin K_{(\varepsilon)}} b_{nk}(j)(\xi_k - \mathfrak{L})\,| + |\sum_{k \in K_{(\varepsilon)}} b_{nk}(j)(\xi_k - \mathfrak{L})\,| \\ &\leq \frac{\varepsilon}{2} \sum_{k \notin K_{(\varepsilon)}} b_{nk}(j) + \sup_k |\,\xi_k - \mathfrak{L}\,| \sum_{k \in K_{(\varepsilon)}} b_{nk}(j). \end{aligned}$$

By the regularity of the method $\mathfrak{B}$, we get

$$\{n \in \mathbb{N} : |(B_j\xi)_n - \mathfrak{L}| \geq \varepsilon\} \subset \{n \in \mathbb{N} : \sum_{k \in K_{(\varepsilon)}} b_{nk}(j) \geq \varepsilon/2M\} \tag{3.1}$$

where $M = \sup_k |\,\xi_k - \mathfrak{L}\,|$. Since $S_{\mathfrak{B}}(\Im)\text{-}\lim \xi = \mathfrak{L}$, (3.1) implies that ξ is $\mathfrak{B}(\Im)$-summable to $\mathfrak{L}$. This completes the proof.

□

We check the converse.

Example 3.2. *Let $\mathbb{N} = \bigcup_r \Lambda_r$ where $\Lambda_r = \{2^{r-1}(2t-1) : t \in \mathbb{N}\}$ are infinite sets and $\Lambda_r \bigcap \Lambda_n = \varnothing$ for $r \neq n$. Let $\Im$ be the collection of all subsets of $\mathbb{N}$ which intersect at most a finite number of Λ_r's. Then $\Im$ is an admissible ideal. Choose*

$$b_{nk}(j) = \begin{cases} \frac{1}{jn} + \frac{1}{j} & \textit{if } n \in \Lambda_1; k = n^2, \\ 1 - \frac{n}{j(n+1)} & \textit{if } n \in \Lambda_1; k = n^2+1, \\ 1, & \textit{.if } n \notin \Lambda_1; k = n^2+1, \\ 0 & \textit{otherwise.} \end{cases}$$

Obviously $\mathfrak{B} \in \mathcal{R}_+$. Now define $\xi = (\xi_k)$ by

$$\xi_k = \begin{cases} k & \text{if } k = n^2, \\ 0 & \text{if } k = n^2 + 1, \\ 1, & \text{otherwise.} \end{cases}$$

Then ξ is $\mathfrak{B}(\Im)$-summable to zero but not $\mathfrak{B}(\Im)$-statistically convergent.

Theorem 3.3. *For an admissible ideal $\Im$, $\mathfrak{B}(\Im^*)\text{-}\lim \xi = \mathfrak{L} \Rightarrow \mathfrak{B}(\Im)\text{-}\lim \xi = \mathfrak{L}$, but generally the converse does not hold.*

Proof. If $\mathfrak{B}(\Im^*)\text{-}\lim \xi = \mathfrak{L}$, then there exists $\mathcal{H} \in \Im$ such that $\mathfrak{K} = \mathbb{N} \backslash \mathcal{H} = \{n_m : n_1 < n_2 < \cdots\} \in \mathcal{F}(\Im)$ and $\mathfrak{B}\text{-}\lim_m \xi_{n_m} = \mathfrak{L}$. But then for each $\varepsilon > 0$ there exists an integer $N > 0$ such that $|(B_j\xi)_{n_m} - \mathfrak{L}| < \varepsilon$ uniformly in j, for all $m > N$. Since

$$\{n_m \in \mathfrak{K} : |(B_j\xi)_{n_m} - \mathfrak{L}| \geq \varepsilon\} \subset \{n_1, n_2, \cdots, n_{N-1}\} \in \Im \ .$$

Hence

$$\{n \in \mathbb{N} : |(B_j\xi)_n - \mathfrak{L}| \geq \varepsilon\} \subseteq \mathcal{H} \cup \{n_m \in \mathfrak{K} : |(B_j\xi)_{n_m} - \mathfrak{L}| \geq \varepsilon\} \in \Im$$

uniformly in j for all $\varepsilon > 0$. Hence $\mathfrak{B}(\Im)\text{-}\lim \xi = \mathfrak{L}$. □

Now we show that the converse does not hold.

Example 3.4. *Let $\Im$ be the class defined in Example 3.2. Define a sequence $\xi_n = \frac{1}{p}$ if $n \in \Lambda_p$. Then $\mathfrak{B}(\Im)\text{-}\lim_n \xi_n = 0$.*

Let $\mathfrak{B}(\Im^*)\text{-}\lim_n \xi_n = 0$. Then there exists $\mathfrak{K} = \{n_m : n_1 < n_2 < \cdots\} \subseteq \mathbb{N}$ such that $\mathfrak{K} \in \mathcal{F}(\Im)$ and $\mathfrak{B}\text{-}\lim_m \xi_{n_m} = 0$. Since $\mathfrak{K} \in \mathcal{F}(\Im)$, there is a set $\mathcal{K} \in \Im$ such that $\mathfrak{K} = \mathbb{N} \setminus \mathcal{K}$. Hence there exists $s \in \mathbb{N}$ such that

$$\mathcal{K} \subset \left(\bigcup_{n=1}^{s} \Lambda_n \right).$$

But then $\Lambda_{s+1} \subset \mathfrak{K}$, and so

$$\xi_{n_m} = \frac{1}{(s+1)} > 0$$

for infinitely many n_m's from $\mathfrak{K}$ which contradicts $\mathfrak{B}\text{-}\lim_m \xi_{n_m} = 0$. Hence the supposition $\mathfrak{B}(\mathcal{I}^*)\text{-}\lim_n \xi_n = 0$ gives a contradiction.

The converse may hold if $\Im$ is a $\mathcal{P}$-ideal, i.e. if for every sequence $(\mathfrak{C}_n)$ of sets from $\Im$ there is $\mathfrak{C} \in \Im$, such that $\mathfrak{C}_n \backslash \mathfrak{C}$ is finite for every n (see [15] and [1]).

Theorem 3.5. *Let $\Im$ be an admissible $\mathcal{P}-$ideal. If $\mathfrak{B}(\Im)\text{-}\lim \xi = \mathfrak{L}$ then $\mathfrak{B}(\Im^*)\text{-}\lim \xi = \mathfrak{L}$.*

Proof. Let $\mathfrak{B}(\Im)\text{-}\lim \xi = \mathfrak{L}$. Then for every $\epsilon > 0$, we have $\{n \in \mathbb{N} : |(B_j\xi)_n - \mathfrak{L}| \geq \varepsilon\} \in \Im$ uniformly in j.

So for every t the sequence $(\mathfrak{C}_t)$ of sets

$$\mathfrak{C}_t = \{n \in \mathbb{N} : |(B_j\xi)_n - \mathfrak{L}| \geq \frac{1}{t}\} \in \Im.$$

Since $\Im$ is a $\mathcal{P}-$ideal, there exists a set $\mathfrak{C} \in \Im$ such that $\mathfrak{C}_t \setminus \mathfrak{C}$ is finite for each t. Let $\mathfrak{K} = \mathbb{N} \setminus \mathfrak{C} = \{n_m\}_{m=1}^{\infty} \in \mathcal{F}(\Im)$. Now for any $\gamma > 0$, there exists $N_\gamma \in \mathbb{N}$ such that $\frac{1}{N_\gamma} < \gamma$, then

$$\mathfrak{C}_N = \left\{ n \in \mathbb{N} : |(B_j\xi)_n - \mathfrak{L}| \geq \frac{1}{N_\gamma} \right\} \in \Im.$$

Therefore the set

$$\left\{n \in \mathbb{N} : |(B_j\xi)_n - \mathfrak{L}| < \frac{1}{N_\gamma}\right\} \in \mathcal{F}(\Im).$$

For $n_m \in \mathfrak{K}$, we have for each $\gamma > 0$

$$|(B_j\xi)_{n_m} - \mathfrak{L}| < \gamma, \forall n_m > N_\gamma, \text{ uniformly in } j.$$

i.e. $\mathfrak{B}(\Im^*)\text{-}\lim \xi = \mathfrak{L}$. This completes the proof. □

Theorem 3.6. *For an admissible ideal $\Im$, if there exists $\mathfrak{K} = \{k_m\}_{m=1}^{\infty} \in \mathcal{F}(\Im)$ and $\delta(\mathfrak{K}) = 1$ such that $\mathfrak{B}\text{-}\lim_m \xi_{k_m} = \mathfrak{L}$, then $\Im(S_{\mathfrak{B}})\text{-}\lim \xi = \mathfrak{L}$ but converse need not be true.*

Proof. Let $\mathfrak{B}\text{-}\lim_m \xi_{k_m} = \mathfrak{L}$, where $\mathfrak{K} = \mathbb{N} \setminus \mathcal{K} = \{k_m\}_{m=1}^{\infty} \in \mathcal{F}(\Im)$, $\mathcal{K} \in \Im$ and $\delta(\mathfrak{K}) = 1$. Hence for each $\varepsilon > 0$,

$$\lim_n \frac{\#\{k_m \leq n : |(B_j\xi)_{k_m} - \mathfrak{L}| \geq \varepsilon\}}{n} = 0, \text{ uniformly in } j.$$

Therefore for each $d > 0$, there exists $N \in \mathbb{N}$ such that $\frac{1}{N} < d$, hence we have

$$\left\{n : \frac{\#\{t \leq n : |(B_j\xi)_t - \mathfrak{L}| \geq \varepsilon\}}{n} \geq d\right\} \subseteq \mathcal{K} \cup \{k_1, k_2, ..., k_N\} \in \Im,$$

i.e. $\Im(S_{\mathfrak{B}})\text{-}\lim \xi = \mathfrak{L}$. This completes the proof. □

Now we check the converse.

Example 3.7. *Let $\Im$ be the class defined in Example 3.2. Choose*

$$b_{nk}(j) = \begin{cases} \frac{1}{jn} + \frac{1}{j} & \text{if } n \in \Lambda_1; k = n, \\ 1 - \frac{n}{j(n+1)} & \text{if } n \in \Lambda_1; k = n^2 \neq 1, \\ 1, & .\text{if } n \notin \Lambda_1; k = n \\ 0 & \text{otherwise.} \end{cases}$$

Obviously $\mathfrak{B} \in \mathcal{R}_+$. Now define $\xi = (\xi_k)$ by

$$\xi_k = \begin{cases} 2 & \text{if } k \notin \Lambda_1, k \text{ is square,} \\ 1, & \text{otherwise.} \end{cases}$$

Then ξ is statistically $\mathfrak{B}(\Im)$-summable to one but not $\mathfrak{B}(\Im)$-summable and hence not $\mathfrak{B}(\Im^*)$-summable nor $\mathfrak{B}(\Im)-$statistically convergent.

Remark. *If $\mathfrak{B}(\Im^*)\text{-}\lim \xi = \mathfrak{L}$, then it is not necessary that $\Im(S_{\mathfrak{B}})\text{-}\lim \xi = \mathfrak{L}$, i.e. $\delta(\mathfrak{K}) = 1$ is necessary. For example, in Example 3.2, we see that ξ is $\mathfrak{B}(\Im)$-summable and $\mathfrak{B}(\Im^*)$-summable to zero but ξ is not statistically $\mathfrak{B}(\Im)$-summable. Moreover there does not exists any set $\mathfrak{K} \in \mathcal{F}(\Im)$ with $\delta(\mathfrak{K}) = 1$ satisfied $\mathfrak{B}\text{-}\lim_m \xi_{k_m} = \mathfrak{L}$.*

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