Journal of Inequalities and Special Functions
ISSN: 2217-4303, URL: www.ilirias.com/jiasf
Volume 13 Issue 1(2022), Pages 14-23.
DOI:10.54379/jiasf-2022-1-2

FUZZY PARAMETERIZED RELATIVE SOFT SETS OVER SOME SEMIGROUPS IN DECISION-MAKING PROBLEMS

PEERAPONG SUEBSAN

Abstract. In this paper, we define fuzzy parameterized relative soft sets over some semigroups and give some its properties. Moreover, we construct a new algorithm for solving some decision-making problems based on fuzzy parameterized relative soft sets over some semigroups.

1. Introduction

The real life problems in computer sciences, economics, environment, medical science, engineering and many other fields are the complex problems. These problems involves data that contain uncertainties. There is a wide range of theories that can be used when dealing with uncertainties in data, such as fuzzy sets [1], intuitionistic fuzzy sets [2], rough sets [3], interval mathematics [4], vague sets [5] and other mathematics tools. Accordingly, in 1999, Molodtsov [6] initiated a new mathematical tool which is soft set theory for modeling uncertainty. He pointed out directions of soft sets for the applications like soft analysis and game theory. Later, Maji *et al.* [7] defined soft subsets and soft super sets, equality of two soft sets. They presented soft binary operations, such as AND, OR, intersection, union and studied De Morgan's Laws of soft sets. Maji *et al.* [8] discussed the application in decision making problems and described the choice value and reduced-soft-sets in decision making problems. After that the soft set theory has been developed by many researchers [9, 10, 11, 12].

In 2001, Maji *et al.* [13] extended the soft sets to fuzzy soft sets. They initiated the concept of fuzzy soft sets and defined fuzzy soft subsets, the intersection and union of fuzzy soft sets over a common universe. In 2007, Roy and Maji [14] used the Comparison table of fuzzy soft sets in the algorithm with decision making problems. Later, Çağman *et al.* [15] introduced the concept of fuzzy parameterized soft sets and their related properties. They proposed a decision making method based on fuzzy parameterized soft sets. Balami and Musa [16] defined relative soft sets and its basic properties which are the generalization of soft sets. Moreover, they discussed its operations such as union, intersection and complement.

The fuzzy soft sets are developed to fuzzy soft semigroups by Yang [17] (2011). He defined fuzzy soft [left, right] ideals over semigroups and fuzzy soft semigroups,

---

2000 *Mathematics Subject Classification.* 03E72, 06C86.

Key words and phrases. Fuzzy parameterized relative soft sets; decision-making problems.

and studied sufficient and necessary conditions for α-level sets, intersection and union of fuzzy soft [left, right] ideals. In 2015, Siripitukdet and Suebsan [18] defined prime, semiprime and strongly prime fuzzy soft bi-ideals over semigroups and presented their properties. In 2018, Suebsan and Siripitukdet [19] studied fuzzy soft bi-ideals over semigroups and its properties. They discussed the mappings on classes of fuzzy soft bi-ideals over semigroups. In 2020, Suebsan and Siripitukdet [20] introduced extended averages of fuzzy soft sets over some semigroups. They constructed the algorithm for solving some decision making problems based on the extended averages of fuzzy soft sets over some semigroups. In 2021, Suebsan [21] presented relative fuzzy soft sets over some semigroups. He constructed the algorithm for solving some decision making problems based on the relative fuzzy soft sets over some semigroups.

Motivated and inspired by the works above, we are interested in the fuzzy parameterized relative soft sets over some semigroups. In this paper, we define fuzzy parameterized relative soft sets over some semigroups and investigate its properties. Moreover, we construct a new algorithm for solving some decision making problems based on fuzzy parameterized relative soft sets over some semigroups.

2. Preliminaries

In this section, we shall give some of basic definitions and results that will be used.

The first concept of soft sets over a universe are introduced by Molodtsov [6].

Definition 2.1. [6] Let U be an initial universe set and let E be a set of parameters. Let $P(U)$ denotes the power set of U and let $\emptyset \neq A \subseteq E$. A pair $(\widehat{F}, A)$ is called a soft set over U, where $\widehat{F}$ is a mapping given by $\widehat{F} : A \to P(U)$.

Note that a soft set $(\widehat{F}, A)$ can be represented by the set of ordered pairs

$$(\widehat{F}, A) = \{(p, \widehat{F}(p)) : p \in A, \widehat{F}(p) \in P(U)\}.$$

Example 2.2. [8] Let $U = \{h_1, h_2, h_3, h_4, h_5\}$ be a set of houses, $E = \{e_1\{\text{beautiful}\}, e_2\{\text{cheap}\}, e_3\{\text{in the green surroundings}\}, e_4\{\text{in good location}\}\}$ be a set of parameters and let $A = \{e_1, e_2, e_3\}$. A soft set $(\widehat{F}, A)$ that describes the "attractiveness of houses for purchase". According to the data collected, the soft set $(\widehat{F}, A)$ is given by

$$(\widehat{F}, A) = \{(e_1, \{h_1, h_3, h_4\}), (e_2, \{h_1, h_2\}), (e_3, \{h_2, h_5\})\}$$

where $\widehat{F}(e_1) = \{h_1, h_3, h_4\}, \widehat{F}(e_2) = \{h_1, h_2\}, \widehat{F}(e_3) = \{h_2, h_5\}$.

Definition 2.3. [1] Let X be a nonempty set. For every set $A \subseteq X$, define its indicator function μ_A,

$$\mu_A(x) = \begin{cases} 1 & \text{if} \quad x \in A, \\ 0 & \text{if} \quad x \notin A. \end{cases}$$

A fuzzy set F is described by its membership function μ_F. For every $x \in X$, this function associates a real number $\mu_F(x)$ in the interval $[0, 1]$. The number $\mu_F(x)$ is interpreted for the point as a degree of belonging x to the fuzzy set F.

The concept of the fuzzy parameterized soft sets are introduced by Çağman et al. [15].

Definition 2.4. [15]Let U be an initial universe, $P(U)$ be the power set of U, E be the set of parameters and X be a fuzzy set over E. A fuzzy parameterized soft set ($fp-$soft set)F_X on the universe U is defined by the set of ordered pairs

$$F_X = \{(\mu_X(x)/x, f_X(x)) : x \in E, f_X(x) \in P(U), \mu_X(x) \in [0,1]\},$$

where the function $f_X : E \to P(U)$ is called an approximate function such that $f_X(x) = \emptyset$ if $\mu_X(x) = 0$, and the function $\mu_X : E \to [0,1]$ is called a membership function of ($fp-$soft set) F_X. The value $\mu_X(x)$ is the degree of importance of the parameter x, and depends on the decision maker's requirements.

Example 2.5. [15] Let $U = \{u_1, u_2, u_3, u_4, u_5\}$ be an initial universe and let $E = \{x_1, x_2, x_3, x_4\}$ be a set of parameters. If $X = \{0.2/x_2, 0.5/x_3, 1/x_4\}$ and

$$\begin{aligned} f_X(x_2) &= \{u_2, u_4\}, \\ f_X(x_3) &= \emptyset, \\ f_X(x_4) &= \{u_1, u_2, u_3, u_4, u_5\} \end{aligned}$$

then the $fp-$soft set F_X is written by

$$F_X = \{(0.2/x_2, \{u_2, u_4\}), (0.5/x_3, \emptyset), (1/x_4, \{u_1, u_2, u_3, u_4, u_5\})\}.$$

The concept of the relative soft sets are introduced by H.M. Balami and I. A. Musa [16].

Let $\{U_i : i \in I\}$ be a collection of universes such that $\bigcap_{i \in I} U_i = \emptyset$ and let $\{E_{U_i} : i \in I\}$ be a collection of set of parameters. $U = P(U_i)$ denotes the power set of $U_i, E = E_{U_i}$ and $A \subseteq E$.

Definition 2.6. [16] A pair $(\widetilde{F}, A)$ is called a relative soft set over U, where $\widetilde{F}$ is a mapping given by $\widetilde{F} : A \to U$. In other words, a relative soft set over U is a parameterized family of subsets of the universe U. For $e \in A, F(e)$ may be considered as the set of $e-$approximate elements of the relative soft set $(\widetilde{F}, A)$. Based on the above definition, any change in the ordering of the universes will produce a different relative soft set.

Example 2.7. [16] Suppose that there are two universe U_1 and U_2, let us consider a relative soft set $(\widetilde{F}, A)$ which describes the "attractiveness of cloths", and "shoes" that Mr. K is considering putting on for a job interview. Let $U_1 = \{C_1, C_2, C_3, C_4, C_5\}$ be the set of cloths and $U_2 = \{S_1, S_2, S_3, S_4\}$ be the set of shoes. Let $E_U = \{E_{U_1}, E_{U_2}\}$ be the collection of sets of decision parameters, where

$$\begin{aligned} E_{U_1} &= \{e_{U_1}, 1 = \text{expensive}, e_{U_1}, 2 = \text{cheap}, e_{U_1}, 3 = \text{beautiful}\}, \\ E_{U_2} &= \{e_{U_2}, 1 = \text{expensive}, e_{U_2}, 2 = \text{made in Italy}, e_{U_2}, 3 = \text{black}\}. \end{aligned}$$

Let $U = P(U_i), E = E_{U_i}$ and $A \subseteq E$ such that $i = 1, 2$. Let

$$A = \{a_1 = (e_{U_1}, 1, e_{U_2}, 1), a_2 = (e_{U_1}, 1, e_{U_2}, 2), a_3 = (e_{U_1}, 2, e_{U_2}, 3), a_4 = (e_{U_1}, 3, e_{U_2}, 2).$$

Suppose that

$$\widetilde{F}(a_1) = (\{C_2, C_3\}, \{S_1, S_4\}),$$
$$\widetilde{F}(a_2) = (\{C_1, C_3\}, \{S_2, S_3\}),$$
$$\widetilde{F}(a_3) = (\{C_1, C_4, C_5\}, \emptyset),$$
$$\widetilde{F}(a_4) = (\{C_2, C_5\}, \{S_2, S_3\}).$$

Then we can see the relative soft set $(\widetilde{F}, A)$ as consisting of the following approximations.

$$(\widetilde{F}, A) = \{(a_1, (\{C_2, C_3\}, \{S_1, S_4\})), (a_2, (\{C_1, C_3\}, \{S_2, S_3\})), (a_3, (\{C_1, C_4, C_5\}, \emptyset)), (a_4, (\{C_2, C_5\}, \{S_2, S_3\})).$$

We can see that each approximation has two part viz; a predicate and an approximate value set. The illustration can logically be explained as follows: for $\widetilde{F}(a_1) = (\{C_2, C_3\}, \{S_1, S_4\}$, if $\{C_2, C_3\}$ is the set of expensive cloths to Mr.K. Then the set of relatively expensive shoes to him is $\{S_1, S_4\}$. It has been shown that relative soft set is a conditional relation.

Suebsan and Siripitukdet [20] defined some semigroups as follows.
Let S be a set and let $\alpha : S \to \mathbb{R}$ be a 1-1 function.
Define operations $\triangle$ and $\bigtriangledown$ on S as follows: For any $x, y \in S$, define

$$x \triangle y = \begin{cases} x & \text{if} \quad \alpha(x) \geq \alpha(y), \\ y & \text{if} \quad \alpha(x) < \alpha(y) \end{cases}$$

and

$$x \bigtriangledown y = \begin{cases} x & \text{if} \quad \alpha(x) < \alpha(y), \\ y & \text{if} \quad \alpha(x) \geq \alpha(y). \end{cases}$$

Then $(S, \triangle)$ and $(S, \bigtriangledown)$ are semigroups. These semigroups are called semigroups induced by a function α and denote S_α. We write $S_\alpha = \{x(\alpha(x))|x \in S\}$.

In real-world problems, we can construct semigroups induced by 1-1 functions.

Example 2.8. [?] A family is looking to purchase a water purifier. Let $S_\alpha = \{o_1(3), o_2(2), o_3(4), o_4(1), o_5(5)\}$ be a set of five water purifiers with a limited time warranty (year) under consideration. Then $(S_\alpha, \triangle)$ is a semigroup.

Example 2.9. [?] A corporation is evaluating the decision of an investment opportunity. Let $S_\alpha = \{o_1(2), o_2(4), o_3(3), o_4(5), o_5(1)\}$ be a set of five investment avenues with risk assessments under consideration, where o_1:Bank Deposit, o_2:Shares, o_3:Mutual Fund, o_4:Stocks, o_5:Government Bonds and the levels of risk are 1:very low, 2:low, 3:moderate, 4:high, 5:very high. Then $(S_\alpha, \bigtriangledown)$ is a semigroup.

3. Fuzzy parameterized relative soft sets over some semigroups

In this section, we define fuzzy parameterized relative soft sets over some semigroups and investigate of their properties.

Let $\{S^i_\alpha : i \in I\}$ be a collection of semigroups such that $\bigcap_{i \in I} S^i_\alpha = \emptyset$, and let $\{E_{S^i_\alpha} : i \in I\}$ a collection of set of parameters. Let $\{X_{S^i_\alpha} : i \in I\}$ be a collection of set of fuzzy sets over E. $S = P(S^i_\alpha)$ denotes the power set of $S^i_\alpha, E = E_{S^i_\alpha}, X = X_{S^i_\alpha}$ and $A \subseteq E$.

Definition 3.1. A fuzzy parameterized relative soft set F_A on the universe S^i_α is defined by the set of ordered pairs

$$F_A = \{(\mu_X(x)/x, f_X(x)) : x \in A, f_X(x) \in S, \mu_X(x) \in [0,1]\},$$

where the function $f_X : A \to S$ is called an approximate function such that

$$f_X(x) = \emptyset \text{ if } \mu_X(x) = 0,$$

and the function $\mu_X : A \to [0,1]$ is called a membership function of F_A. The value $\mu_X(x)$ is the degree of importance of the parameter x, and depends on the decision maker's requirements.

Example 3.2. Suppose that there are three universe S^1_α, S^2_α and S^3_α. Let us consider a fuzzy parameterized relative soft set F_A which describes the condition of some states in a country. The company with enough capital is considering for the location of its manufacturing industries. Let $S^1_\alpha = \{C_1(150), C_2(180), C_3(200)\}$ be a set of states with availability of land with a distance (km) from the state to head office of the company under consideration. Let S^1_α be a semigroup with a binary operation $\triangledown$ defined by Table 1.

TABLE 1. The multiplication table of a semigroup S^1_α

$\triangledown$	C_1	C_2	C_3
C_1	C_1	C_1	C_1
C_2	C_1	C_2	C_2
C_3	C_1	C_2	C_3

Let $S^2_\alpha = \{C_4(100), C_5(120), C_6(140)\}$ be a set of states with availability of labour with a distance (km) from the state to head office of the company under consideration. Let S^2_α be a semigroup with a binary operation $\triangledown$ defined by Table 2.

TABLE 2. The multiplication table of a semigroup S^2_α

$\triangledown$	C_4	C_5	C_6
C_4	C_4	C_4	C_4
C_5	C_4	C_5	C_5
C_6	C_4	C_5	C_6

Let $S^3_\alpha = \{C_7(160), C_8(170), C_9(190)\}$ be a set of states with availability of raw materials with a distance (km) from the state to head office of the company (km) under consideration. Let S^3_α be a semigroup with a binary operation $\triangledown$ defined by Table 3.

TABLE 3. The multiplication table of a semigroup S^3_α

$\triangledown$	C_7	C_8	C_9
C_7	C_7	C_7	C_7
C_8	C_7	C_8	C_8
C_9	C_7	C_8	C_9

Let $E_{S_\alpha} = \{E_{S_\alpha^1}, E_{S_\alpha^2}, E_{S_\alpha^3}\}$ be a collection of the set of parameters related to the above universes, where

$$E_{S_\alpha^1} = \{e_{11} = \text{peaceful state}, e_{12} = \text{commercial state}, e_{13} = \text{armed robbery state}, e_{14} = \text{good weather state}, e_{15} = \text{densely populated state}\},$$
$$E_{S_\alpha^2} = \{e_{21} = \text{power state}, e_{22} = \text{harsh weather state}, e_{23} = \text{violent state}, e_{24} = \text{densely populated state}\},$$
$$E_{S_\alpha^3} = \{e_{31} = \text{accessible state}, e_{32} = \text{good weather state}, e_{33} = \text{power state}, e_{34} = \text{sparsely populated state}\}.$$

Let $X = \{X_{S_\alpha^1}, X_{S_\alpha^2}, X_{S_\alpha^3}\}$ be a collection of the set of fuzzy set over E, where

$$X_{S_\alpha^1} = \{0.2/e_{11}, 0.5/e_{12}, 0.1/e_{13}, 0.5/e_{14}, 1/e_{15}\},$$
$$X_{S_\alpha^2} = \{0.5/e_{21}, 0.3/e_{22}, 0.2/e_{23}, 1/e_{24}\},$$
$$X_{S_\alpha^1} = \{0.6/e_{31}, 0.5/e_{32}, 1/e_{33}, 0.9/e_{34}\}.$$

Let $S = P(S_\alpha^i), E = E_{S_\alpha^i}$ and $A \subseteq E$ such that $i = 1, 2, 3$. Let

$$A = \{a_1 = (0.2/e_{11}, 0.5/e_{21}, 0.6/e_{31}), a_2 = (0.5/e_{12}, 1/e_{24}, 0.5/e_{32}), a_3 = (0.5/e_{14}, 0.2/e_{23}, 0.9/e_{34}), a_4 = (0.1/e_{13}, 1/e_{24}, 0.9/e_{34}), a_5 = (1/e_{15}, 0.5/e_{21}, 0.6/e_{31}), a_6 = (0.2/e_{11}, 1/e_{24}, 0.9/e_{34})\}.$$

Suppose that

$$f_X(a_1(0.2, 0.5, 0.6)) = (\{C_2, C_3\}, \{C_5\}, \{C_7, C_8\}),$$
$$f_X(a_2(0.5, 1, 0.5)) = (\{C_1, C_2\}, \{C_6\}, \{C_9\}),$$
$$f_X(a_3(0.5, 0.2, 0.9)) = (\{C_1, C_2, C_3\}, \{C_6\}, \emptyset),$$
$$f_X(a_4(0.1, 1, 0.9)) = (\{C_2\}, \{C_6\}, \emptyset),$$
$$f_X(a_5(1, 0.5, 0.6)) = (\{C_3\}, \{C_5\}, \{C_7, C_8\}),$$
$$f_X(a_6(0.2, 1, 0.9)) = (\{C_2, C_3\}, \{C_6\}, \emptyset).$$

Then we can see the fuzzy parameterized relative soft set F_A as consisting of the following approximations.

$$F_A = \{(a_1(0.2, 0.5, 0.6), (\{C_2, C_3\}, \{C_5\}, \{C_7, C_8\})),$$
$$(a_2(0.5, 1, 0.5), (\{C_1, C_2\}, \{C_6\}, \{C_9\})),$$
$$(a_3(0.5, 0.2, 0.9), (\{C_1, C_2, C_3\}, \{C_6\}, \emptyset)),$$
$$(a_4(0.1, 1, 0.9), (\{C_2\}, \{C_6\}, \emptyset)),$$
$$(a_5(1, 0.5, 0.6), (\{C_3\}, \{C_5\}, \{C_7, C_8\})),$$
$$(a_6(0.2, 1, 0.9), (\{C_2, C_3\}, \{C_6\}, \emptyset)).$$

We can see that each approximation has three part viz; a predicate and an approximate value set. The illustration can logically be explained as follows: for $f_X(a_1(0.2, 0.5, 0.6)) = (\{C_2, C_3\}, \{C_5\}, \{C_7, C_8\})$, if $\{C_2, C_3\}$ is the set of peaceful states to the company then the states it can relatively obtain regular electric power supply from to is $\{C_5\}$. If $\{C_2, C_3\}$ is the set of peaceful states to the company and $\{C_5\}$ is the set of state it can obtain regular electric power supply then the set of relative accessible state to it is $\{C_7, C_8\}$. It has been shown that fuzzy parameterized relative soft set is a conditional relation.

Definition 3.3. For two fuzzy parameterized relative soft sets F_A and G_B over S^i_α, F_A is called a fuzzy parameterized relative soft subset of G_B if

(1) $A \subseteq B$ and $\mu_{X_F}(x) \leq \mu_{X_G}(x)$ for all $x \in A$ and
(2) for all $e_{ij} \in a_k, (e_{ij}, F_{e_{ij}}) \subseteq (e_{ij}, G_{e_{ij}})$, where $a_k \in A, k \in \{1, 2, ..., n\}$, $i \in \{1, 2, ..., m\}, j \in \{1, 2, ..., r\}$.

This relationship is denoted by $F_A \widetilde{\subseteq} G_B$. this case G_B is called a fuzzy parameterized relative soft superset of F_A.

Definition 3.4. Two fuzzy parameterized relative soft sets F_A and G_B over S^i_α are said to be equal if F_A is a fuzzy parameterized relative soft subset of G_B and G_B is a fuzzy parameterized relative soft subset of F_A.

Example 3.5. Consider Example 3.2, let

$$\begin{aligned} A =& \{a_1 = (0.2/e_{11}, 0.5/e_{21}, 0.6/e_{31}), a_2 = (0.5/e_{12}, 1/e_{24}, 0.5/e_{32}),\\ & a_3 = (0.5/e_{14}, 0.2/e_{23}, 0.9/e_{34}), a_4 = (0.1/e_{13}, 1/e_{24}, 0.9/e_{34}),\\ B =& \{a_1 = (0.2/e_{11}, 0.5/e_{21}, 0.6/e_{31}), a_2 = (0.5/e_{12}, 1/e_{24}, 0.5/e_{32}),\\ & a_3 = (0.5/e_{14}, 0.2/e_{23}, 0.9/e_{34}), a_4 = (0.1/e_{13}, 1/e_{24}, 0.9/e_{34}),\\ & a_5 = (1/e_{15}, 0.5/e_{21}, 0.6/e_{31}), a_6 = (0.2/e_{11}, 1/e_{24}, 0.9/e_{34})\}. \end{aligned}$$

Clearly, $A \subseteq B$ and $\mu_{X_F}(x) \leq \mu_{X_G}(x)$ for all $x \in A$. Let F_A and G_B be two fuzzy parameterized relative soft sets over the same S^i_α such that

$$\begin{aligned} F_A = \{ & (a_1(0.2, 0.5, 0.6), (\{C_2, C_3\}, \{C_5\}, \{C_7, C_8\})),\\ & (a_2(0.5, 1, 0.5), (\{C_1, C_2\}, \{C_6\}, \{C_9\})),\\ & (a_3(0.5, 0.2, 0.9), (\{C_1, C_2, C_3\}, \{C_6\}, \emptyset)),\\ & (a_4(0.1, 1, 0.9), (\{C_2\}, \{C_6\}, \emptyset)).\\ G_B = \{ & (a_1(0.2, 0.5, 0.6), (\{C_2, C_3\}, \{C_5\}, \{C_7, C_8\})),\\ & (a_2(0.5, 1, 0.5), (\{C_1, C_2\}, \{C_6\}, \{C_9\})),\\ & (a_3(0.5, 0.2, 0.9), (\{C_1, C_2, C_3\}, \{C_6\}, \emptyset)),\\ & (a_4(0.1, 1, 0.9), (\{C_2\}, \{C_6\}, \emptyset)),\\ & (a_5(1, 0.5, 0.6), (\{C_3\}, \{C_5\}, \{C_7, C_8\})),\\ & (a_6(0.2, 1, 0.9), (\{C_2, C_3\}, \{C_6\}, \emptyset)). \end{aligned}$$

Hence $F_A \widetilde{\subseteq} G_B$.

Now, we present some operations of two relative fuzzy soft sets over some semigroups as below.

Definition 3.6. The union of two fuzzy parameterized relative soft sets F_A and G_B over S^i_α, denoted by $F_A \widetilde{\cup} G_B$, is defined by $F_A \widetilde{\cup} G_B = H_C$ if

(1) $\mu_{X_F \widetilde{\cup} X_G}(x) = \max\{\mu_{X_F}(x), \mu_{X_G}(x)\}$ for all $x \in C$ and
(2) for all $\epsilon \in C = A \cup B$,

$$H(\epsilon) = \begin{cases} F(\epsilon), & \text{if} \quad \epsilon \in A - B,\\ G(\epsilon), & \text{if} \quad \epsilon \in B - A,\\ F(\epsilon) \cup G(\epsilon), & \text{if} \quad \epsilon \in A \cap B. \end{cases}$$

Proposition 3.7. *If F_A, G_B and H_C are three fuzzy parameterized relative soft sets over S^i_α then*
(1) $F_A \widetilde{\cup} F_A = F_A$ *and*
(2) $(F_A \widetilde{\cup} G_B) \widetilde{\cup} H_C = F_A \widetilde{\cup} (G_B \widetilde{\cup} H_C)$.

Definition 3.8. The intersection of two fuzzy parameterized relative soft sets F_A and G_B over S^i_α, denoted by $F_A \widetilde{\cap} G_B$, is defined by $F_A \widetilde{\cap} G_B = H_C$ if
(1) $\mu_{X_F \widetilde{\cap} X_G}(x) = \min\{\mu_{X_F}(x), \mu_{X_G}(x)\}$ for all $x \in C$ and
(2) for all $\epsilon \in C = A \cup B$,

$$H(\epsilon) = \begin{cases} F(\epsilon), & \text{if } \epsilon \in A - B, \\ G(\epsilon), & \text{if } \epsilon \in B - A, \\ F(\epsilon) \cap G(\epsilon), & \text{if } \epsilon \in A \cap B. \end{cases}$$

Proposition 3.9. *If F_A, G_B and H_C are three fuzzy parameterized relative soft sets over S^i_α then*
(1) $F_A \widetilde{\cap} F_A = F_A$ *and*
(2) $(F_A \widetilde{\cap} G_B) \widetilde{\cap} H_C = F_A \widetilde{\cap} (G_B \widetilde{\cap} H_C)$.

Proposition 3.10. *If F_A, G_B and H_C are three fuzzy parameterized relative soft sets over S^i_α then*
(1) $F_A \widetilde{\cup} (G_B \widetilde{\cap} H_C) = (F_A \widetilde{\cup} G_B) \widetilde{\cap} (F_A \widetilde{\cup} H_C)$ *and*
(2) $F_A \widetilde{\cap} (G_B \widetilde{\cup} H_C) = (F_A \widetilde{\cap} G_B) \widetilde{\cup} (F_A \widetilde{\cap} H_C)$.

4. Applications

In this section, we apply the concept of the fuzzy parameterized relative soft sets over some semigroups in the new algorithm for solving in some decision-making problems.

Let $\{S^i_\alpha : i \in I\}$ be a collection of semigroups such that $\bigcap_{i \in I} S^i_\alpha = \emptyset$, and let $\{E_{S^i_\alpha} : i \in I\}$ a collection of set of parameters. Let $\{X_{S^i_\alpha} : i \in I\}$ be a collection of set of fuzzy sets over E. $S = P(S^i_\alpha)$ denotes the power set of S^i_α, $E = E_{S^i_\alpha}$, $X = X_{S^i_\alpha}$ and $A \subseteq E$.

Now, we present an application of fuzzy parameterized relative soft sets over some semigroups in a decision-making problem.

Algorithm

Step 1. Input the fuzzy parameterized relative soft set F_A.
Step 2. Compute the score value T_A. T_A is defined by

$$T_A = \frac{1}{|A|} \sum_{p \in A} \mu_A(p) \chi_{f_A(p)}(u),$$

where $|A|$ is the number of elements of A, $f_A(p)$ is the subset determined by the parameter p and

$$\chi_{f_A(p)}(u) = \begin{cases} 1 & \text{if } u \in f_A(p), \\ 0 & \text{if } u \notin f_A(p). \end{cases}$$

Step 3. The decision is C_k. If $C_k = \max T_A$. If C_k has more than one object then we choose C_k corresponding to operation of the semigroup.

The following example using the new algorithm in a decision making problem. The concept is similar to the Example 3.2.

Example 4.1. From Example 3.2, suppose that there are three universe S^1_α, S^2_α and S^3_α. Let us consider a fuzzy parameterized relative soft set F_A which describes the condition of some states in a country. The company with enough capital is considering for the location of its manufacturing industries. Let $S^i_\alpha, i \in \{1,2,3\}$ be semigroups with a binary operation $\bigtriangledown$ defined by Table 1, Table 2 and Table 3 in the Example 3.2.

Step 1. Let F_A be a fuzzy parameterized relative soft set over S^i_α as consisting of the following approximations.

$$\begin{aligned} F_A = \{&(a_1(0.2,0.5,0.6),(\{C_2,C_3\},\{C_5\},\{C_7,C_8\})),\\ &(a_2(0.5,1,0.5),(\{C_1,C_2\},\{C_6\},\{C_9\})),\\ &(a_3(0.5,0.2,0.9),(\{C_1,C_2,C_3\},\{C_6\},\emptyset)),\\ &(a_4(0.1,1,0.9),(\{C_2\},\{C_6\},\emptyset)),\\ &(a_5(1,0.5,0.6),(\{C_3\},\{C_5\},\{C_7,C_8\})),\\ &(a_6(0.2,1,0.9),(\{C_2,C_3\},\{C_6\},\emptyset)). \end{aligned}$$

Step 2. Compute the score value T_A of a fuzzy parameterized relative soft set F_A over S^i_α.

TABLE 4. The score value of fuzzy parameterized relative soft set F_A

S^i_α	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
$a_1(0.2,0.5,0.6)$	0	1	1	0	1	0	1	1	0
$a_2(0.5,1,0.5)$	1	1	0	0	0	1	0	0	1
$a_3(0.5,0.2,0.9)$	1	1	1	0	0	1	0	0	0
$a_4(0.1,1,0.9)$	0	1	0	0	0	1	0	0	0
$a_5(1,0.5,0.6)$	0	0	1	0	1	0	1	1	0
$a_6(0.2,1,0.9)$	0	1	1	0	0	1	0	0	0
$T_A = \frac{1}{\|A\|}\sum_{p\in A}\mu_A(p)\chi_{f_A(p)}(u)$	0.17	0.25	0.32	0.00	0.17	0.53	0.20	0.20	0.08

Step 3. The decision is C_k. Hence the company is selected C_3 for the states with availability of land and is selected C_6 for the states with availability of labour. Since C_7 and C_8 which have the maximum score value, the company is selected C_7 corresponding to operation of the semigroup in Table 3 for the states with availability of raw materials.

5. CONCLUSIONS

In this paper, fuzzy parameterized relative soft sets over some semigroups are introduced. Moreover, the new algorithm for multiple evaluation in decision-making problems based on fuzzy parameterized relative soft sets over some semigroups are presented. The proposed algorithm is practical for solving some decision-making problems.

Acknowledgments. The author would like to thank the anonymous referee for his/her comments that helped us improve this article.

References

[1] L.A. Zadeh, *Fuzzy sets*, Information and Control **8** (1965) 338–353.

[2] K. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems **20** (1986) 87–96.

[3] Z. Pawlak, *Rough sets*, International Journal of Information Technology and Computer Sciences **11** (1986) 341–356.

[4] M.B. Gorzalzany, *A method of inference in approximate reasoning based on interval-valued fuzzy sets*, Fuzzy Sets and Systems **21** (1987) 1–17.

[5] W.L. Gau and D.J. Bueher, *Vague sets*, IEEE Transaction System Man Cybernet **23(2)** (1993) 610–614.

[6] D. Molodtsov, *Soft set theory-First results*, Computer and Mathematics with Applications **37** (1999) 19–31.

[7] P.K. Maji, R. Biswas and A.R. Roy, *An application of soft sets in a decision making problem*, Computer and Mathematics with Applications **44** (2002) 1077–1083.

[8] P.K. Maji, R. Biswas and A.R. Roy, *Soft set theory*, Computer and Mathematics with Applications **45** (2003) 555–562.

[9] H. Aktaş and N. Çağman, *Soft sets and soft groups*, Information Sciences **177** (2007) 2726–2735.

[10] M.I. Ali, F. Feng, X. Liu, W.K. Min and M. Shabir, *On some new operations in soft sets theory*, Computer and Mathematics with Applications **57** (2009) 1547–1553.

[11] M.I. Ali, M. Shabir and M. Naz, *Algebraic structures of soft sets associated with new operations*, Computer and Mathematics with Applications **61** (2011) 2647–2654.

[12] A. Kharal and B. Ahmad, *Mapping on soft classes*, New Mathematics Natural Computation **7(3)** (2011) 471–481.

[13] P.K. Maji, R. Biswas and A.R. Roy, *Fuzzy soft set*, The Journal of Fuzzy Mathematics **9(3)** (2011) 589–602.

[14] A.R. Roy and P.K. Maji, *A fuzzy soft set theoretic approach to decision making problems*, Journal of Computational and Applied Mathematics **203** (2007) 412–418.

[15] N. Çağman, F. Çitak and S. Enginoğlu, *FP- soft set theory and its applications*, Annals of Fuzzy Mathematics and Informatics **2(2)** (2011) 219–226.

[16] H.M. Balami and I.A. Musa, *On relative soft set*, International Journal of Scientific Research and Management **1(9)** (2013) 494–506.

[17] C.F. Yang, *Fuzzy soft semigroups and fuzzy soft ideals*, Computer and Mathematics with Applications **44** (2011) 255–261.

[18] M. Siripitukdet and P. Suebsan, *On fuzzy soft bi-ideals over semigroups*, Songklanakarin Journal of Science and Technology **37(5)** (2015) 237–249.

[19] P. Suebsan and M. Siripitukdet, *The mappings on classes of fuzzy soft bi-ideals over semigroups*, Azerbaijan Journal of Mathematics **8(2)** (2018) 43–59.

[20] P. Suebsan and M.Siripitukdet, *Applications of fuzzy soft sets over some semigroups based on extended averages*, IAENG International Journal of Computer Science, **47(4)** (2020) 816–822.

[21] P. Suebsan *On relative fuzzy soft sets over some semigroups in decision-making problems*, Journal of Discrete Mathematical Sciences and Cryptography, **24(1)** (2021) 209–222, DOI:10.1080/09720529.2020.1845466

Peerapong Suebsan
Department of Mathematics, School of Science, University of Phayao, Postal 56000 Phayao, Thailand
E-mail address: `peerapong.su@up.ac.th`